# A Hydrological Event Model of the Tollgate Wetland<sup>\*</sup>

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May 3, 2000

#### Abstract

The Clean Water Act of 1972 as amended in 1987 requires states to control the flow of pollution from storm water into local rivers. In most cases, this requirement is carried out through installation of an expensive system of pipes and pumps. The Tollgate Drainage District, Ingham County, Michigan, examined here, uses a more economical alternate approach: water from the district flows into an artificially created and maintained wetland designed to naturally clean the district's storm water.

This report examines the mechanisms at work in the Tollgate wetland, proposes a unit hydrograph and model for water flow, describes and discusses a simulation of the system designed by the authors based on these proposals, and gives conclusions and recommendations for further study. Appendices contain reference material on the relevant hydrological phenomena and a user's manual for the simulation model.

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\*Work done for the Ingham County Drain Commission, Pat Lindemann commissioner, from January to May 2000, for MSU course MTH 844, under the direction of C. R. MacCluer, professor of mathematics.

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# 1 Introduction

Federal legislation mandates control of the flow of pollution from storm water into rivers. For small communities, the cost of traditional implementation of this requirement through installation of pipes and pumps can be prohibitive. An alternate approach is to drain storm water into a constructed wetland. This approach is used in the Tollgate Drainage District of Ingham County, Michigan, depicted below in Figure 1.



Figure 1: Sketch of Tollgate wetland [1].

The primary design goal for the system was as a storage reservoir: it is designed to be large enough, in conjunction with overflow to the neighboring golf course, to hold the runoff from a 100-year storm, a storm of magnitude such that on average it will occur only once in 100 years [1].

However, storage is not the only point of interest in the Tollgate wetland. Questions of interest not addressed in the original Tollgate engineering work include the following:

- How does water move through the system as the total volume handled increases?
- How do conservative and nonconservative pollutants move through the system?
- What special maintenance needs might the system be expected to have, and given these, what is the system's expected lifespan as designed?

This report provides part of the answer to the first question, by constructing an *event model* of the Tollgate wetland detention system. This model predicts an answer to the following question: Given a set of initial conditions in the system and a pattern of precipitation, what response, in terms of a short-term water flow pattern, can be expected within the system?

This report commences with a review of the hydrology literature on constructed wetlands. Decomposition of the problem follows, first with an examination of precipitation and resulting runoff, then with a study of how water enters the system. These principles are then applied directly to the Tollgate system via an event model simulation, and results and conclusions are given, along with recommendations for further study. The first appendix contains definitions and references on hydrological phenomena that are likely to be useful for further work; a second appendix acts as a user's guide to the event model.

# 2 Literature Review

The following literature review surveys articles related to analysis and design of constructed wetlands. The initial search turned up four categories of promising articles. Some of these turned out to be helpful, but most turned out to lack the desired depth and precision necessary for mathematical modeling.

The articles were evaluated as follows:

• Water Science and Technology

The abstracts for these articles [2, 3, 4, 5] arouse interest, but when the articles were retrieved from the library, all were found to be much too short and superficial to be of much use. The given case studies were described qualitatively, not quantitatively, and few formulas were used.

[3] is typical. According to the abstract, "The influence of three typical outlet structures on the hydrologic regime of a hypothetical wetland was investigated using a continuous simulation approach." This is interesting: understanding the outlets between sections of the Tollgate wetland is important. However, the article itself fails to give details of how the simulation was accomplished or the equations used. Thus, the results cannot be directly applied to the Tollgate wetland.

As a result, we concluded that these articles are not useful for our project.

• Proceedings of the ASCE Wetlands Engineering River Restoration Conference 1998

The articles [6, 7, 8, 9, 10] in [11] all appear interesting from the abstracts. However, for our purposes, they suffer from much the same flaws as the *Water Science and Technology* articles.

[10] is also typical. The abstract states, "This approach provides...new insight into seasonal and long-term performance of constructed wetlands." Indeed, the article does provide some qualitative insight and presents some data in graphical form, but it is primarily oriented toward those having large amounts of sample data and those interested in designing new systems, rather than those analyzing existing systems without benefit of sample data.

• Ecological Engineering

The MSU library does not carry this journal. We did not request inter-library loans for these articles [12, 13, 14, 15, 16].

• Other articles

Each of the articles in this category is from a different journal. We retrieved all of them that were in MSU's library, which turned out to be only four articles.

One article [17] was of interest primarily for its list of references. For instance, it was from this list of references that we originally learned that "constructed wetland" is the standard term for an man-made wetland.

Another article [18] turned out to be of more intrinsic interest. Some of the formulas in this article turned out to be of use and are cited as such in this report. Other formulas, such as those for predicting frequency of rain events, could be of use in extensions of this work.

The remaining articles [19, 20] were not of use, for the same reasons given as for the articles from *Water Science and Technology* above.

### 3 Runoff

When precipitation occurs over a watershed, some of the precipitated water is retained within the watershed, above or below the soil. The rest, called *runoff*, flows (directly or indirectly) into storm sewers [21, p. 204–206]. In the Tollgate watershed, runoff travels to the Tollgate wetland.

The amount of runoff from a given storm depends on the area of the watershed, the amount of precipitation, and the imperviousness of the watershed area. The total amount of runoff can thus be represented as

$$R = CpA,\tag{1}$$

where C is a coefficient representing soil permittivity taken as an average over the area of the watershed, p is the precipitation depth, and A is the watershed's area. [18]

A hydrograph is "a graph of stage or discharge versus time" [21, p. 117]. In the Tollgate watershed, the hydrograph of interest is runoff from the watershed into the grit chamber and the detention pond. In Figure 1 on page 2, this hydrograph is measured at the bottom of the diagram where the inflow divides between the grit chamber and the detention pond.

A unit hydrograph is a defining hydrograph for a system given a unit amount of precipitation over the basin, such as 1 inch. As the duration of rainfall approaches zero, keeping the amount of precipitation constant, the unit hydrograph becomes an instantaneous unit hydrograph (**IUH**). Given a rainfall pattern f(t) and an IUH  $i_e$ , runoff q at time t can be calculated via the convolution

$$q(t) = \int_0^t f(\tau) i_e(t-\tau) d\tau$$
(2)

Hydrographs and unit hydrographs can be constructed from actual data measured during particular events, or they can be devised synthetically using heuristic methods. The former is preferable: "[U]nit hydrographs derived by synthetic and transposition methods seem to be of limited value" [21, p. 225]. However, the latter methods have the advantage of not requiring as much data as the former methods. Since measured event data is not available for the Tollgate wetland, heuristics will be used here.

Several methods have evolved for synthetically constructing a unit hydrograph based on minimal actual data from a watershed. Many of these methods have the following two-part structure:

- 1) Heuristically determine values for  $t_p$ ,  $Q_p$ , and  $t_b$  (defined later).
- 2) Fit a curve to the points determined by  $t_p$ ,  $Q_p$ , and  $t_b$ .

These two parts are discussed separately below.

#### 3.1 Parameter Estimation

The shape of a theoretical unit hydrograph can be determined by three parameters:  $t_p$ , the time of maximum runoff or peak time,  $Q_p$ , the maximum runoff flow rate, and  $t_b$ , the time base or time for runoff flow to recede to zero. The time origin t = 0 is taken to be the start time of precipitation. [22, p. 175–179]

The literature gives numerous ways to estimate values for these parameters. One common method, due to Snyder [22, p. 206–208], makes use of simple measurements of the watershed's sewer system. Snyder's equations give peak time, in hours, as

$$t_p = C_t (LL_c)^{0.3} \tag{3}$$

where L is the length of the main stream from outlet to divide, in miles,  $L_c$  is the distance from the outlet to a point on the stream nearest to the centroid of the basin, also in miles, and  $C_t$  is a coefficient varying between 1.8 and 2.2 depending on the slope of the basin.

Snyder calculates peak flow as

$$Q_p = \frac{640C_pA}{t_p} \tag{4}$$

where A is drainage area in square miles,  $C_p$  is a coefficient ranging from 0.56 to 0.69, and 640 is a conversion factor to cause  $Q_p$  to be in ft<sup>3</sup>/s.

Lastly, Snyder's time base is

$$t_b = 3 + 3\frac{t_p}{24} \tag{5}$$

where  $t_p$  is peak time as calculated above. [21, p. 223]

#### 3.2 Curve Fitting

The parameters determined using the techniques of the previous section specify three points: (0, 0),  $(t_p, Q_p)$ , and  $(t_b, 0)$ . Any number of techniques can be used to fit a curve to these three points. Two such techniques are examined below.

The simplest approach draws straight lines among the points:

$$f(t) = \begin{cases} \frac{Q_p}{t_p} t, & 0 \le t \le t_p, \\ Q_p - \frac{Q_p}{t_b - t_p} (t - t_p), & t_p \le t \le t_b, \\ 0, & t_b \le t \end{cases}$$
(6)

A problem with this approach is that it does not necessarily produce a function having unit area under its curve. The more refined approach below avoids this difficulty.

[22] suggests use of a two-parameter gamma distribution function, given by

$$f(t) = \frac{t^{\alpha} e^{-t/\beta}}{\beta^{\alpha+1} \Gamma(\alpha+1)}$$
(7)

where  $0 < t < \infty$ ,  $\alpha > -1$  is dimensionless, and  $\beta > 0$  has the same units as t. Figure 2 shows the gamma distribution function's shape for selected parameter values.



Figure 2: Shape of the gamma distribution function  $f(t) = \frac{t^{\alpha}e^{-t/\beta}}{\beta^{\alpha+1}\Gamma(\alpha+1)}$  for peak time  $t_p = 1$  and several peak values  $Q_p$ . Note that the area under the gamma distribution is always 1, regardless of parameters.

For our purposes, the gamma distribution function has two helpful properties: first, its shape is the same as observed hydrographs, and second, there is unit area under its curve for all values of  $\alpha$  and  $\beta$ .

Since we want the maximum value of f(t) to occur at  $t = t_p$ , we can find one constraint on  $\alpha$  and  $\beta$  by finding f(t)'s maximum. Setting the derivative of f(t) equal to zero, we obtain

$$\alpha t^{\alpha-1} e^{-t/\beta} - \frac{t^{\alpha}}{\beta} e^{-t/\beta} = 0$$

Simplifying,

$$t^{\alpha-1}\left(\alpha-\frac{t}{\beta}\right)=0.$$

Therefore,

$$t = \alpha \beta = t_p. \tag{8}$$

A second constraint on  $\alpha$  and  $\beta$  can be found by setting  $Q_p = f(\alpha\beta)$ :

$$f(\alpha\beta) = \frac{(\alpha\beta)^{\alpha}e^{-\alpha}}{\beta^{\alpha+1}\Gamma(\alpha+1)} = \frac{\alpha^{\alpha+1}}{t_p e^{\alpha}\Gamma(\alpha+1)}$$

Let A be the area of the basin and  $C_v A$  be a unit volume. Then

$$Q_p = \frac{C_v A \alpha^{\alpha+1}}{t_p e^{\alpha} \Gamma(\alpha+1)} = \frac{C_v A}{t_p} \phi(\alpha).$$
(9)

According to [22, p. 205],  $\phi(\alpha)$  can be estimated as the cubic

$$\alpha = 0.045 + 0.5\phi + 5.6\phi^2 + 0.3\phi^3. \tag{10}$$

Combining Equations 9 and 10, we arrive at

$$\alpha = 0.045 + 0.5 \left(\frac{Q_p t_p}{C_v A}\right) + 5.6 \left(\frac{Q_p t_p}{C_v A}\right)^2 + 0.3 \left(\frac{Q_p t_p}{C_v A}\right)^3.$$
(11)

Using Equations 8 and 11, we can calculate  $\alpha$  and  $\beta$  given  $t_p$  and  $Q_p$ .

The gamma distribution function has an infinite tail, so it cannot be made to pass through  $(t_b, 0)$ . However, it approaches the x-axis arbitrarily closely as x approaches infinity, so in practice the unit hydrograph function g(t) can be written in a piecewise fashion:

$$g(t) = \begin{cases} f(t), & 0 \le t < t_b \\ 0, & t \ge t_b \end{cases}.$$
 (12)

### 4 Recirculation Pump

In the Tollgate wetland, water in the grit chamber is pumped to the waterfall, flowing into the deadwood swamp, as shown in Figure 1 on page 2. Therefore, the operation of the pump directly affects the circulation of water in the system. In this section, we examine the pump's operating characteristics.

There are actually two pumps in the Tollgate wetland. These pumps are identical in specifications. Their operation is guided by a set of water level sensors, which are positioned such that the following relationships are satisfied:

$$l_{off} < l_{1on} < l_{2on} \tag{13}$$

When the level l in the grit chamber is minimal, such that  $l < l_{1on}$ , neither pump runs. When the level rises, such that  $l_{1on} \leq l < l_{2on}$ , one pump operates. If the level then falls such that  $l < l_{off}$ , the pump stops. Contrariwise, if the level continues rising such that  $l \geq l_{2on}$ , both pumps run until the level again falls such that  $l < l_{off}$ , at which point both pumps stop.



Figure 3: Theoretical and actual pump rates for pump in Tollgate system.

The flow rate of a pump depends primarily on the presented *head*, or the vertical distance between the water level on the inlet and outlet sides of the pump. For the particular pump used in the Tollgate wetland, regression on sampled points from supplied pump curves yielded the following theoretical flow rate approximation

$$Q_{p}(h) = 21.45 - (7.821 \times 10^{-3})h - (7.129 \times 10^{-4})h^{2} + (1.388 \times 10^{-6})h^{3}, \tag{14}$$

where h is head in feet, giving a result in hundreds of gallons per minute [23]. Figure 3 shows this theoretical pump rate as a dashed line. Note that the theoretical output of two identical pumps is twice that of a single pump of the same type.

Because of flow resistance, the actual pump rate is lower than the theoretical pump rate. The actual pump rate at a given head h can be found via an iterative process, described below.

- 1) Figure an initial guess for  $(H_L)_f$ , the head loss due to friction in the pump output pipe. Zero is acceptable as an initial guess if no better value is known.
- 2) Calculate  $Q_p$  for  $h + (H_L)_f$ , using (14).
- 3) Calculate water velocity v from  $Q_p$ :

$$v = \frac{Q_p}{\pi r^2} \tag{15}$$

where r is the radius of the pipe (5 in).

4) Calculate *Re*, the *Reynolds number* for the flow, given *v*:

$$Re = \frac{2v\rho r}{\mu} \tag{16}$$

where  $\rho$  is the density of water (62.32 lb/ft<sup>3</sup>) and  $\mu$  is the viscosity of water (2.39 lb/h·ft). [24, p. 2.3]

5) Calculate f, the Fanning friction factor for the flow, given Re [24, p. 2.9]:

$$f = \frac{0.3164}{Re^{0.25}} \qquad \text{for} \quad Re < 10^5 \tag{17}$$

$$f = 0.0032 + \frac{0.221}{Re^{0.237}}$$
 for  $10^5 < Re < 3 \times 10^6$  (18)

6) Calculate  $(H_L)_{f1}$ , the head loss due to friction [24, p. 2.9]:

$$(H_L)_{f1} = f\left(\frac{L}{2r}\right) \left(\frac{v^2}{2g}\right) \tag{19}$$

where L is the length of the pipe and g is the force of gravity (32.17 ft/s<sup>2</sup>).

7) If  $|(H_L)_{f1} - (H_L)_f| < \epsilon$ , where  $\epsilon$  is the maximum acceptable error, then accept the  $Q_p$  calculated most recently in step 2 as a good approximation real value. Otherwise, set  $(H_L)_f \leftarrow (H_L)_{f1}$  and repeat from step 2.

When the above process is applied to a range of heads, a function giving real pump rate in terms of theoretical pump rate is obtained. This function is shown as a solid line in Figure 3 for the cases of one and two pumps in operation.

#### 5 Actual Figures

Many of the preceding sections dealt with wetlands in general, with little regard for the specific case of the Tollgate wetland. However, actual figures for the Tollgate wetland must be used to obtain much insight. This section enumerates Tollgate parameters and their sources.

The Tollgate watershed is 210 acres in size [1]. Snyder's L, the main stream channel from outlet to divide, is 0.9219 miles. Snyder's  $L_C$ , the main stream channel from outlet to the point nearest the watershed centroid, is 0.9583 miles [25]. Synder's  $C_t$  and  $C_p$  are both estimated as 0.3 [22, p. 208]. Runoff permittivity C is estimated at 43% [26].

The grit chamber is box-shaped, 10 ft by 7.5 by 7 ft, with a 10 in by 7.5 ft by 2 ft unusable filled volume. Its floor has an elevation of 838 ft. It is connected to the pump chamber by a 24 in pipe of unknown length. The pump chamber is cylindrical with 6 ft diameter and 11 ft height with a minimum elevation of 834 ft. The pump's outlet in the waterfall is at an elevation of 859.5 ft. [27]. Therefore, the usable volume of the grit chamber and pump chamber together is approximately 824 ft<sup>3</sup>, ignoring the connecting pipe.

The deadwood swamp has a surface area of 12,668 ft<sup>2</sup> [25]. The shallow pond has a surface area of 12,128 ft<sup>2</sup>. Both exhibit 0.5 ft difference in depth between low-water and high-water conditions [27].

The shortest distance through the sand-peat filter, from inlet to outlet, is 305 ft. Following an orthogonal path, the distance is 350 ft [25].

The ditch has area 5,528 ft<sup>2</sup> at elevation 845 ft and area 15,810 ft<sup>2</sup> at its high elevation, 847 ft [25]. Using a trapezoidal approximation to its cross-sectional area, its maximum capacity is 21,338 ft<sup>3</sup>.

The detention poind has surface area 273,450 ft<sup>2</sup> with a 3.5 ft maximum normal change in depth [27]. The product, 957,075 ft<sup>3</sup>, is a lower bound on its maximum expected capacity.

## 6 Simulation

Figure 4 is a block diagram of the flow of water in the Tollgate wetland. Each block represents one stage that water passes through. Our knowledge of the individual elements in the system, as discussed in earlier sections, along with this illustration of how the elements works together, allows a simulation of the system to be constructed.



Figure 4: Block diagram of water flow in Tollgate wetland. Solid lines mark direction of gravity flow; dashed lines denote pumps; dotted lines indicate raised pipes.

The simulation based on this level of knowledge makes a number of simplifying assumptions. Only the rainfall pattern, hydrograph, grit chamber, pump, and detention pond are explicitly modeled. The system of ponds is arbitrarily reduced to a one-hour delay between the grit chamber and the detention pond. Water is assumed not to flow from the detention pond into the grit chamber. (See the Discussion, page 12, for justification of the two final assumptions.)

This model is suitable only as an event model, because it does not account for water leaving the wetland via evaporation, transpiration, or infiltration. Modifying the model to deal with these phenomena would be a logical extension; some relevant facts can be found in the appendix.

The model does not account for the ponds in the system. Earlier versions of this model attempted to include these effects in the model, but it turned out that too little data was available to nontrivially simulate them. Obviously, learning more about the ponds and the streams connecting them is important for future work.

The model is implemented with a software package called Simulink for Matlab 5.3 by Mathworks, Inc. This program allows control systems to be built schematically, in a hierarchical manner, and their behavior to be simulated. Matlab and Simulink files for the Tollgate model are included with this report, and their usage is documented in the appendix.

Two types of measurements appear in the system: water flow rates, in  $ft^3/min$ , and water volumes, in  $ft^3$ . The underlying theme of the model is that integrating a water flow rate over a period of time gives a water volume.

The input to the system is provided by a user-specified *hyetograph* or rainfall pattern; e.g., 1 inch of water over a 1-hour period. This flow passes into the grit chamber, or if the grit chamber is full, into the detention pond. Thus, when it is not bypassed by high-flow conditions, the grit



Figure 5: Control system model for the Tollgate system.



Figure 6: Control system model for grit chamber and pump.

chamber acts as a buffer between the watershed and the wetland.

Figure 5 shows the top-level Simulink model for the wetland system. The input hydrograph is connected to the input divider, which routes water to the grit chamber, unless it is full, in which case it goes to the detention pond directly. The grit chamber is connected to the wetland system, arbitrarily represented as a one-hour delay, whose output connects to the detention pond. A virtual oscilloscope records the hyetograph and characteristics of the system's response.

The model for the grit chamber and pump is illustrated by Figure 6. In this model, the integral of the flow rate into the chamber produces the volume of water in the chamber. This volume is used to calculate the water level and the pump head, i.e., the vertical distance that the pump or pumps must lift water. The water level in turn determines the number of pumps operating; the head determines the rate at which the pumps operate. An amplifier converts the pump rate from gal/min to  $ft^3/min$  for conformability of units with the remainder of the system. Finally, the outflow is fed back into the input in order to ensure consistency of the grit chamber volume.

The grit chamber model uses a submodel, shown in Figure 7, to convert grit chamber volume to water level and pump head. The piecewise linear formulas used in this submodel are easily derived



Figure 7: Control system model for conversion between grit chamber volume and pump head.



Figure 8: Control system model for input divider.

from the grit chamber dimensions specified on page 5.

The input divider model compares the grit chamber volume to its capacity. Water is routed into the grit chamber unless it is full, in which case the water is passed into the detention pond directly. For hysteresis' sake a relay is used instead of an exact comparator; if this is not done, Simulink iterates indefinitely.

### 7 Discussion

Figure 9 on page 13 shows the simulated response of the Tollgate system to three sample events. Each response is shown as five graphs against time (in minutes). The graphs, from top to bottom, represent:

- a. Hyetograph or rainfall pattern, in in/h.
- b. Hydrograph or wetland inflow, in  $ft^3/min$ .
- c. Amount of water passed through grit chamber during event, in ft<sup>3</sup>.
- d. Amount of water passed into detention pond, bypassing grit chamber, during event, in ft<sup>3</sup>.
- e. Total amount of runoff, in  $ft^3$ .

The three sample events are (1) one inch of rain over a one-hour period, (2) 0.5 inch, then 1.5 inch, then 0.5 inch of rain in successive half-hour periods, and (3) 0.2 inch of rain over one hour.

Examination of simulation results (1) or (2) reveals a surprising result: the water passing into the detention pond is about the same as the total runoff volume. This means that in a rainstorm, the grit chamber fills quickly, causing much of the storm runoff to flow directly into the detention pond.



Figure 9: Results of simulation of three sample events. From top to bottom and left to right, the events are (1) one inch of rain over a one-hour period, (2) 0.5 inch, then 1.5 inch, then 0.5 inch of rain in successive half-hour periods, and (3) 0.2 inch of rain over one hour.

One of the stated purposes of the grit chamber is to remove large pieces of debris from incoming water, so water bypassing the grit chamber would seem to be undesirable and run counter to the system's design goals.

Conservative calculations can verify that the simulation matches the Tollgate model in this case. Suppose that 1 inch of rain falls on the watershed over a 1-hour period. The runoff amount R is approximately 43% of the total precipitation volume, as (1):

$$R = CpA = 0.43 \times 1$$
 in  $\times 210$  acre  $= 3.3 \times 10^5$  ft<sup>3</sup>.

Suppose further that the runoff is discharged uniformly across a 3-hour period, noting that the unit hydrograph for the system predicts a shorter discharge time with a high peak. Then, if the pump rate from the grit chamber is considered to be the maximum, 4300 gal/min (see Figure 3), then for the grit chamber the ratio k of water inflow versus outflow is

$$k = \frac{3.3 \times 10^5 \text{ ft}^3/3 \text{ h}}{4300 \text{ gal/min}} = 3.17$$

Since the volume of the grit chamber is just 824  $\text{ft}^3$  (see page 9) (2.6 fewer orders of magnitude) it quickly fills under these conditions.

Simulation result (3) shows a result closer to that originally expected, for which most of the runoff passes through the grit chamber on its way into the system. The rainfall pattern of 0.2 inch over a hour to produce this result was empirically determined.

### 8 Conclusion

This report has examined the properties of the Tollgate constructed wetland in Ingham County, Michigan, beginning with a review of the literature on constructed wetlands. We have constructed a synthetic hydrograph for the Tollgate watershed, figured the behavior of the grit chamber and pump, and constructed a event simulation.

The results of the simulation showed that, in a storm of magnitude larger than 0.2 in/h, most of the storm runoff is not filtered through the grit chamber, but rather bypasses it, directly flowing into the detention pond. This is undesirable due to the useful filtering of large particles that the grit chamber provides.

An obvious next step in studying the Tollgate system is to attempt to confirm or refute these results. We wish to suggest that direct observation of the system following a rainstorm may be the simplest and most effective way to do this: in a moderate rainstorm, does most of the runoff in fact pass directly into the detention pond?

More research is needed to obtain a truly accurate event model of the Tollgate system. The most evident need for further information is for a better understanding of how water passes from one pond to another in the system. Of the existing research, the most suspect is the hydrograph. This could be improved by obtaining a more accurate unit hydrograph through measurement of the runoff from one or more actual rainstorms.

Beyond an event model, other questions pose themselves. Is the Tollgate system effective for cleaning water, or only in storing it? The authors noted an offensive organic smell at some times, especially in proximity to the sand/peat filter; does this indicate a problem in the system? Finally, extensive research might begin to answer the question of the expected effective lifetime of the Tollgate system.

#### 8.1 Acknowledgements

The authors wish to express their thanks to Randy Abbott, Don Cuthbert, Pat Lindemann, Katie Palmer, Dave Solberg, and Karen Wayland of the Ingham County Drain Commissioner's office, Roger Wallace of the MSU department of civil engineering, Tom Volkening of the MSU engineering library, John LeFevre of Fishbeck, Thompson, Carr, and Huber, Philip FitzSimons of FitzSimons Automation, fellow students Md. Nazrul Islam and Asad Qazi, and of course our faculty supervisor, Charles MacCluer of the MSU department of mathematics. Without the valuable contributions of all of these people, this work would not have been possible.

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## Appendix A Hydrological Factors

In building a seasonal or annual model of the Tollgate system, rather than an event model, additional factors must be considered. The complete list of factors must include the following:

- **Inflow and outflow.** All of the blocks are interconnected. Gravity causes flow in one direction naturally, and a pump system keeps water circulating against the flow of gravity. Moreover, in several places, above-ground pipes allow water to flow when water surpasses a given level.
- **Evaporation.** Water exposed to air and sun evaporates, reducing water volume. Rate of evaporation can be determined in a number of ways depending on the known information. Factors that can be considered include vapor pressure, water surface temperature, air temperature, wind speed, and humidity [21]. Evaporation is reduced or eliminated for water surface covered by vegetation.
- **Transpiration.** Transpiration is water taken up by a plant's root system and discharged into the air as vapor. Transpiration rate depends especially on plant type and water available to plants.
- **Precipitation.** Rain contributes directly to water volume. Snow does not contribute to water volume until melting, which may be immediate or delayed.
- **Infiltration.** When water passes from a body of water into the ground, this is infiltration. Infiltration rate depends on soil type, *head* or height of water within the soil, and pressure from water overlaying soil.

The body of this report addresses only inflow and outflow and precipitation. However, future studies may involve the other hydrological processes above. The following sections reports on some research done on these topics that will prove useful in future Tollgate work.

#### A.1 Evaporation

Numerous empirically derived formulas exist for estimating evaporation from water surface. These formulas differ in the assumptions made and the knowledge required for application. Let's examine two of these.

Meyer's equation is

$$E = C(e_0 + e_a) \left(1 + \frac{W}{10}\right) \tag{20}$$

where E is daily evaporation in inches, W is wind velocity in miles per hour measured 25 ft above the water surface, C is an empirically determined pan coefficient (about 0.36 for an ordinary lake),  $e_0$  is saturation vapor pressure at water surface temperature in inches of mercury (in. Hg), and  $e_a$ is vapor pressure of air in in. Hg [22, p. 90].

Saturation vapor pressure can be approximated using the formula

$$e_s = 2.749 \times 10^8 \exp\left(\frac{-4278.6}{T + 242.79}\right) \tag{21}$$

where  $e_s$  is in millibars and T is in degrees Celsius [22, p. 18]. This formula can be used to compute  $e_0$  directly.  $e_a$  is calculated as the product of saturation vapor pressure of air and relative humidity.

Another method for measuring evaporation makes use of *evaporation pans*. These pans are typically 4 ft in diameter, 10 inches deep, and rest 12 inches above the ground. Evaporation from these pans can be related to evaporation from bodies of water by multiplication by a constant factor, usually between 0.70 and 0.75. This factor is consistent on a yearly basis but may change significantly from month to month.

Penman derived the following formula for estimation of daily evaporation E:

$$E_L = C_p \left( \frac{Q_n \Delta + \gamma E_a}{\Delta + \gamma} \right) \tag{22}$$

where  $C_p$  is an appropriate pan constant,  $\Delta = \frac{de_s}{dT}\Big|_{T=T_a}$  with  $T_a$  as the air temperature,  $E_a$  is daily pan evaporation in millimeters,  $\gamma$  is net radiant energy in the same units as E, and  $\gamma = \frac{0.61p}{1000}$  with p as atmospheric pressure in millibars [22, p. 92–93]. Some examples of empirically determined pan constants are found in [21, p. 151].

#### A.2 Transpiration

Most of the water absorbed by the roots of plants is transmitted through them by evaporation through *stomates*, or tiny holes in their leaves. Since transpiration is simply evaporation from plants' leaves, all the same factors that affect evaporation (as discussed in the previous section) also affect transpiration. [22, p. 96–97]

Under normal conditions, transpiration rate varies little among different types of plants within a single biome. Among biomes, desert plants transpire more slowly than temperate species. When available soil moisture is limited, species with deep roots transpire more than those with shallow roots. [21, p. 156–157]

It is difficult to measure transpiration rates directly in the field. In laboratory conditions, transpiration can be measured using tanks and losses measured by weighing, but such data cannot be applied directly to field conditions. [22, p. 97].

One method for estimating seasonal transpiration for a particular crop is the Blaney-Criddle method:

$$U = k_s B \tag{23}$$

where U is the total water consumption over the season in inches and  $k_s$  is an empirical coefficient applicable to a particular crop and area. The value B is calculated as

$$B = \sum \frac{tp}{100} \tag{24}$$

where t is mean monthly temperature in degrees Fahrenheit and p is "average daytime hours in percentage of the year." Thus, the monthly consumption p in inches can be found using the formula

$$u = \frac{ktp}{100} \tag{25}$$

given the same variables as above. [22, p. 98–99].

#### A.3 Infiltration

Infiltration is the water movement from the soil surface into the soil. Once water is applied to soil surface, gravity causes water to move down through many layers of the subsurface. Two major subsurface zones are divided by the *water table*. The water table is an irregular surface, defined as the level of the water where the hydrostatic pressure is equal to atmospheric pressure. A surface body of water, such as a river or a pond, is an extension of the water table where it extends over the surface of the ground. The zone above the water table is the *vadose* zone or zone of aeration. The zone below the water table is the *phreatic zone* or zone of saturation. Water in the phreatic zone is also called ground water [28, 21].

The potential energy per unit mass of water is capillary potential. More precisely, capillary potential is the work required to move a unit mass of water from a reference plane to any point in the soil column. Since water will move upward by capillarity without any external work, the capillary potential is negative. This suggests that the capillary potential  $\psi$  is the product of gravity g and the height y (a negative value):  $\psi = gy$ .

Given these definitions, we can discuss movement of the soil moisture. As water is applied to the soil surface, capillary pores at the surface are filled and the intake capacity is lowered, causing the infiltration rate to decline. Infiltration rate decreases gradually in homogeneous soil until the zone of aeration is saturated. Movement of the moisture in soil is governed by the equation

$$q = -K_w \frac{\partial \Lambda}{\partial x} \tag{26}$$

where q is flow per unit time through unit area normal to the direction of flow, x is the distance along the line of flow,  $K_w$  is conductivity, and  $\Lambda$  is potential. The equation implies that flow is from regions of high potential to regions to lower potential. Determining the conductivity  $K_w$  is difficult but it is known to increase with moisture content and decrease with pore size. Thus, capillary movement decreases as soil dries and as soil becomes more fine grained.

Movement of moisture in soil leads us to movement of ground water. As mentioned earlier, ground water is water below the water table. Its movement is governed by Darcy's law: v = ks, where v is velocity of flow, s is the slope of the hydraulic gradient, and k is a coefficient. Discharge q is the product of area A and velocity. The effective area is the total area times porosity p, which is in turn the ratio of the pore volume to the total volume of the formation. Thus,

$$q = vAp = kpAs = K_pAs \tag{27}$$

where coefficient  $K_p$  is called the *coefficient of permeability* or the hydraulic conductivity.  $K_p$  depends on properties of the fluid and medium.

## Appendix B Simulation Model User's Guide

The simulation model for the Tollgate wetland is provided as a set of files for The Mathworks' Matlab 5.3 with Simulink option. The model was developed using, and the instructions below reflect the use of, the Unix version of Matlab, but versions for other operating systems should be similar.

The model is segmented into two parts: a Matlab code module for converting a hydrograph into a hydrograph, and a Simulink model to simulate the system response to a hydrograph. The former module is invoked from the Matlab command line. To use it, first be sure that the current directory contains the Tollgate .m files: hydro.m, iuh.m, make\_rain.m, s\_hydrograph.m, and unit\_hydrograph.m. Use the ls (list files) command to confirm. If this is not the case, Matlab's cd command can be used to change directories.

The hydrograph synthesis function, hydro, takes a single argument, rainpat, that specifies the pattern of rainfall. rainpat takes the form of an *n*-row by 2-column matrix. The first column in each row specifies a length of time, in hours, and the second column specifies the amount of rainfall, in inches, during the corresponding time period.

The hydro function returns a pair of structures specifying the hydrograph and the hyetograph, respectively. The primary use of these return values is as input to the Tollgate Simulink model. For this purpose, the returned structured must be assigned to Matlab workspace variables h and r.

The following examples show how the hydrographs for the three sample events illustrated in Figure 9 can be reconstructed.

- 1) [h, r] = hydro ([1 1]);
- 2) [h, r] = hydro ([.5 .5; .5 1.5; .5 .5]);
- 3) [h, r] = hydro ([1 .2]);

The Simulink model is invoked from Simulink. To launch Simulink, type simulink at the Matlab command prompt. Shortly an additional window should pop up. In that window, select "Open..." from the "File" menu, and select the provided file finalmodel.mdl from the list of files.

The model shown in Figure 5 on page 11 should appear. An oscilloscope readout resembling Figure 9 on page 13 should also be displayed. If not, double-click on the oscilloscope icon in the model window.

Once a hydrograph has been created using syntax similar to that given above, the model can be run. To do this, choose "Start" on the "Simulation" menu. Depending on the speed of your computer, it may take a while for the simulation to run. When the simulation is complete, the computer beeps.

The oscilloscope window can be used to examine the system response. The graphs in the window are in the order explained on page 12. If some of the graphs go off the charts, click on the "Autoscale" button on the oscilloscope's toolbar to rescale them. Other buttons on the toolbar can be used to zoom in on particular parts of the graphs, to print the graphs, or to change properties of the oscilloscope window.

If the chosen rainfall pattern is much longer than 1.5 h, it may be desirable to simulate the system for longer than 3 h. If so, two settings must be adjusted: the simulation runtime and the oscilloscope display time. Use the "Parameters" item on the "Simulation" menu of the model system to change the former, and the "Properties" button on the oscilloscope's toolbar to change the latter. Normally, both should be set to the same values. Also, please note that both settings are specified in minutes.