Routelet Placement for Multipath Transport (Prelim. version)

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Abstract

In this paper, we address the placement of relay service agents (routelets) in the Internet to assist multipath transport protocols; these protocols are designed to achieve better network utilization and fairness by exploiting path diversity in the Internet. We identify three different routelet deployment scenarios, provide LP formulations for routelet placement in each of the scenarios, and prove that the placement problem is NP-hard in two of the scenarios. We provide rounding algorithms with provable properties for a subset of the scenarios, and compare their performance through simulations on several BRITE topologies of varying scales. We observe that our rounding algorithm leads to a 2-5 fold increase in bandwidth utilization compared to the default single path routing. This closely matches the performance of the optimal LP throughput in the several topologies tested.

I. Introduction

Several multipath routing and congestion control protocols (mTCP [31], Han et al. [11] Kelly et. al. [15], Key et al. [16] and Harp [18]) have been proposed recently for better network resource allocation and utilization. The key idea of these proposals is to send packets from a source to a destination through multiple Internet paths; the sending rate on each path is determined by a congestion control algorithm that ensures better network-wide bandwidth utilization and fairness of allocation. Commercial products such as Asankya [1] enable high-quality real-time content using the above idea of multiple paths for packet transport. All of these works are motivated by the observations that (1) Internet today has significant path diversity [26], i.e. given any two communicating end-hosts in the Internet, there exist potentially many independent or partially overlapping paths between them, (2) today's routing infrastructure often limits the traversal of packets between a source and a destination to a *single* path, and (3) the routing underlay adapts too slowly to fluctuating network conditions such as failures and bandwidth availability, thereby causing load imbalance [4] on several paths in the Internet that makes applications run at suboptimal performance.

A key commonality in the multipath protocol proposals is the presumption that some nodes in the Internet (such as overlay routers [22] or diversified routers [30] or stepping-stone routers [16]) can provide relay services for packets thereby allowing packets to take many alternate paths; the agents providing relay service on each node are henceforth called *routelets* [6]. However, few works address the problem of *where in the Internet* to place the routelets to be most effective for bandwidth utilization. In this paper, we address this exact problem:

Given a graph G that represents the routers and links in the Internet, a subset of the routers that are capable of hosting the routelets, and a set of sender-receiver pairs that transmit and receive data, where should we place k routelets to maximize bandwidth utilization?.

A handful of works address a similar relay placement problem for providing resilience to path failures between a source and a destination [12], [25], [29], or for improving end-throughput by breaking an end-to-end TCP connection into a series of shorter-loop connections [19], [25]. The key idea in these works is to have an alternate path through a relay node that shares minimum number of links possible with the default underlay path. However, these solutions do not directly apply to bandwidth maximization because

Scenario	LP	Hardness	Rounding	Rounding Proof
a	√	Open	√	√
b	√	√	√	Open
c	√	√	Open	Open

TABLE I
CONTRIBUTIONS AND OPEN PROBLEMS.

for this purpose, the overlay paths can share as many links as required, as long as the shared links do not become bottlenecks. This difference makes the placement problem for bandwidth maximization more complex.

In this paper, we consider the routelet placement problem for bandwidth maximization under three deployment scenarios. We make three contributions. Firstly, we formalize the three scenarios of the problem, and formulate them in a constraint optimization framework. We prove that the placement problem in two of the scenarios is NP-hard. Second, we develop rounding algorithms for two scenarios and prove for one scenario that the objective (of bandwidth maximization) remains within a $log\ n$ factor of the optimal fractional solution, while using no more than a factor $O(log\ n)$ more routelets than the fractional solution. Finally, we show through simulations with several BRITE [21] topologies of varying node populations, node degree, and link bandwidths that our rounding algorithms perform very close to the optimal solutions. We summarize our contributions and some interesting; open problems in Table I.

The rest of the paper is organized as follows. Section II describes the deployment dcenarios and defines the problem statements. Section III presents the LP formulations for the deployment scenarios. In Section IV, we present rounding algorithms for two scenarios and prove certain desirable properties. In Section V, we present simulation results to demonstrate the efficacy of our rounding algorithms on several network topologies. Section VI discusses the related work, and Section VII concludes. We prove the hardness of scenario b and give the idea behind the hardness result for scenario c in Appendix.

II. PROBLEM FORMULATION

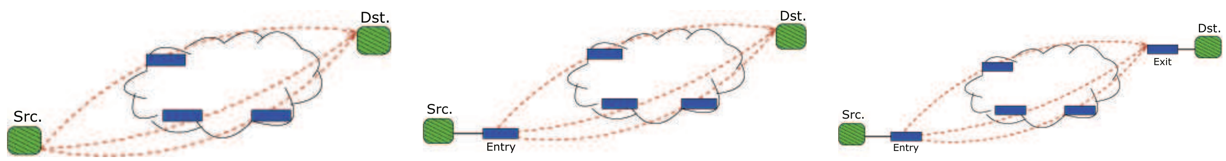
We envision that multipath routing and congestion control protocols will be realized in the Internet in three steps (and timescales). First, a subset of nodes in the Internet called *relay nodes* are deployed for hosting relay services. Second, for a given set of source and destination (SD) pairs, a certain number of *relay service agents* or *routelets* are placed on a subset of the relay nodes to let the SD pairs use alternate paths in the Internet. Finally, using the set of paths enabled by the routelets, multipath routing protocols determine the appropriate packet sending rates on each path between a given SD pair. While node placement is a coarse timescale operation (say days to months), routelet placement can happen at a granularity of minutes to hours, and multipath routing and rate control work at a fine granularity of round-trip times between a source and a destination.

In this paper, our focus is on the second step: our objective is to determine the placement of routelets in the Internet such that any two communicating end-hosts can exploit path diversity by exchanging packets through the routelets. In the rest of this section, we discuss the deployment scenarios and formally define the problem statements.

A. Deployment Scenarios

We explore the routelet placement problem under three deployment scenarios (see Figure 1).

1) **Scenario (a)** In the scenario shown in Figure 1(a), both the source and the destination nodes are modified to choose alternate paths through the routelets for routing packets. In particular the source node maintains a list of routelets to which it can send packets. Packets are then forwarded to the



(a) (b) (c)

Fig. 1. Deployment Scenarios

destination by the routelets. Since using different paths for packets can lead to packet reordering, the destination performs the necessary reordering.

In this scenario, the routelets do not maintain any state for routing packets. Packets are encapsulated by the source nodes, hence the actual destination address is present in each packet. The routelets just remove the encapsulation and forward the packets towards the destination. As a result, the functionality of the routelets is simple. The disadvantage of this approach, however, is twofold. First, the sources and destinations need to be modified, which makes the solution much harder to deploy compared to a solution that does not need modifications. Second, even though the senders are modified to include the functionality of making the packets take different routes, the actual set of routelets that are to be used between a source and a destination can keep changing with time due to traffic fluctuations. The frequency of updating each sender with the set of good routelets constantly strikes a tradeoff between inefficient use of relay resources, and the generation of unnecessary network traffic for updates.

2) **Scenario (b)** Figure 1(b) shows a deployment scenario that addresses one of the major drawbacks of Scenario (a). In this scenario, the functionality of choosing different paths is offloaded to an *entry* routelet, thereby leaving the sender unmodified. The entry routelet can act so for several senders and hence allows for sharing information about available relay routelets. Such sharing reduces the amount of traffic generated to keep the set of good relay routelets updated. The destination, however, still requires modifications to either avoid or tolerate the effects of out-of-order packets.
3) **Scenario (c)** The scenario in Figure 1(c) obviates the need to change either the source or the destination nodes, thereby making the solution readily deployable. This scenario employs an exit routelet to which packets are sent from the relay routelets; the exit routelet reorders the packets before forwarding to the destination, thereby masking off the effects of utilizing multiple paths.

Scenarios (b) and (c) require extra functionality and also maintain state in the entry and exit routelets than Scenario (a). However, they are more attractive for deployment since they require minimum modification to the sources and destinations.

We make the following assumptions in this paper.

- The underlay routing gives only a single default path for any pair of source and destination nodes in the network.
- Each path has a single bottleneck link.
- All the SD-pairs are known and active.
  We can also extend the results to the case when a good model for the percentage of time that they are active is given, but we do not address this extension in this paper.

B. Problem Statements

Here we will formalize the problem definitions for all of the scenarios studied, with bandwidth utilization between the SD-pairs as the optimization criterion.

1) **Scenario a** Given a graph G that represents the nodes and links in the Internet, a subset of the nodes that are capable of hosting the routelets, and a set of SD-pairs, where should we place at

most k routelets to maximize the bandwidth utilization between the SD-pairs, assuming the paths an SD-pair can utilize to send packets are either a default path or a Source-Relay-Destination (S-R-D) path)?
2) **Scenario b** The same problem as in (a), but now the paths can either be the default path or Source-Entry-Relay-Destination (S-E-R-D) paths, with the additional constraint that the entry routelet needs to be on the default path.
3) **Scenario c** The same problem as in (a), but now the paths can either be the default or Source-Entry-Relay-Exit-Destination (S-E-R-E-D) paths, with the constraint that the entry routelet needs to be on the default path, and the default path can only be used if the exit routelet is located on it.
4) **No routelets** Given a graph G that represents the nodes and links in the Internet and a set of SD-pairs, what is the maximum bandwidth that can be achieved using just the default paths? (the MAX-FLOW problem)

III. LP Formulation

In this section, we formulate the routelet placement problem in each scenario as a set of linear constraints and objectives. We begin by formulating the base-case: a scenario in which there are no routelets, and the total bandwidth between sources and destinations is just governed by the underlay routing. This formulation will serve as a baseline for subsequent formulations of the routelet placement problems, and also for comparing the efficacy of employing routelets for maximizing bandwidth utilization.

A. Scenario with no routelets

Let f_{sd} represent the flow (or bandwidth[1]) between a source s and a destination d on the default route (chosen by the underlay routing scheme). Our objective for this scenario is to represent the total flow between all pairs of sources and destinations. Hence the total flow is represented as

$$\sum_{sd} f_{sd} \tag{1}$$

$$\tag{2}$$

Observe that the flow between s and d is restricted by the bottleneck link capacity on the default route between them. This constraint is represented as

$$\sum_{e \in P_{sd}} f_{sd} \leq c_e, \forall e \in E \tag{3}$$

where e represents an edge and P_{sd} represents the set of edges on the default route between s and d.

Finally, the flow between any sd can not be negative, which is represented as

$$f_{sd} \geq 0 \tag{4}$$

B. Scenario (a)

Our idea in this scenario is to place a relay routelet to enable packets to take alternate paths, in addition to the direct routes (or paths) chosen by the default routing scheme. We introduce the following variables.

1) S is the set of all sd pairs.
2) f_{sd} is the flow from s to d on the default route.
3) $f_{sd,m}$ is the flow from s to d via relay routelet m.
4) $P_{sd,m}$ is the path (i.e., a set of edges) corresponding to the flow $f_{sd,m}$.
5) $M_k \in \{0, 1\}$ is an indicator variable which represents whether node k is selected to host a relay routelet or not.

[1]In the rest of the section, we use the word flow to represent bandwidth.

6) $\text{B}_{sd,k,m}$ denotes the capacity of the bottleneck link on the path $P_{sd,k,m}$.
7) $\mathcal{K}$ is the number of routelets to be deployed.

The LP formulation is represented as:

$$\max \quad \sum_{sd,m} f_{sd,m} + f_{sd} \tag{5}$$

$$\text{s.t.}$$

$$\sum_{k} M_k \leq \mathcal{K} \tag{6}$$

$$\forall sd, m \quad f_{sd,m} \leq M_m \times \mathcal{B}_{sd,m} \tag{7}$$

$$\forall k, e \in E \quad \sum_{sd} \mathcal{I}(e \in P_{sd,k}) \times f_{sd,k} \leq M_k \mathcal{C}_e \tag{8}$$

$$\forall e \in E \quad \sum_{sd,k} \mathcal{I}(e \in P_{sd,k}) \times f_{sd,k} \leq \mathcal{C}_e \tag{9}$$

$$f_{sd,m} \geq 0 \tag{10}$$

$$M_k \geq 0 \tag{11}$$

The objective function in (5) represents the sum of bandwidths achieved on default paths as well as paths through relay routelets between all source-destination pairs. Constraint (6) bounds the number of routelets placed in the network. Constraint (7) bounds the total flow through a routelet on node m to be at most the bottleneck bandwidth on the path through m, whereas constraint (8) restricts the the total flow passing through a routelet on node m and through a given edge e to be at most the edge capacity. We multiply both of these by an indicator of whether a routelet is actually present on node m. Finally (9) restricts the sum of flows passing through each edge to be at most the edge capacity.

C. Scenario (b)

In Scenario (b), some of the total routelets will be used as entry routelets. Hence, the formulation requires the following notation.

1) S is the set of all SD pairs (given).
2) f_{sd} is the flow from s to d on the default route.
3) $f_{sd,k,m}$ is the flow from s to d redirected by the entry routelet on node k via relay routelet on node m.
4) $P_{sd,k,m}$ is the path (i.e., a set of edges) corresponding to the flow $f_{sd,k,m}$.
5) $M_k \in \{0, 1\}$ is an indicator variable which represents whether node k is selected to host a relay routelet or not.
6) $\mathcal{B}_{sd,k,m}$ denotes the capacity of the bottleneck link on the path $P_{sd,k,m}$.
7) $\mathcal{K}$ is the number of routelets to be deployed.
8) $\mathcal{I}(e \in P_{sd,k,m})$ is an indicator (known apriori), which is set to 1 if edge e is contained in $P_{sd,k,m}$.
9) $\mathcal{C}_e$ represents the total capacity of edge e.
10) $D_{sd,m}$ is only defined if the routelet on node m is on the default path from s to d. This variable indicates if this routelet acts as "entry" point for this SD-pair.

The LP formulation for Scenario (b) is as follows.

$$\max \quad \sum_{sd,m,k} f_{sd,m,k} + f_{sd} \tag{12}$$

$$\text{s.t.} \quad \sum_{k} M_k \leq \mathcal{K} \tag{13}$$

$$\forall sd \quad \sum_{m} D_{sd,m} \leq 1 \tag{14}$$

$$\forall sd, m \quad D_{sd,m} \leq M_m \tag{15}$$

$$\forall sd, k \quad f_{sd,m,k} \leq D_{sd,m} \times \mathcal{B}_{sd,m,k} \tag{16}$$

$$\forall sd, k \quad f_{sd,m,k} \leq M_k \times \mathcal{B}_{sd,m,k} \tag{17}$$

$$\forall e \in E \sum_{sd,m,k} \mathcal{I}(e \in P_{sd,m,k}) f_{sd,m,k} \leq \mathcal{C}_e \tag{18}$$

$$f_{sd,m,k} \geq 0 \tag{19}$$

$$M_k \geq 0 \tag{20}$$

$$D_{sd,m} \geq 0 \tag{21}$$

The constraint (13) bounds the number of routelets used. The constraints (16) and (17) allow flows to pass only via eligible paths (in which the first routelet is a valid entry routelet, and the second is a valid relay routelet). Constraint (18) is the edge capacity constraint. The constraint (15 allows for at most one entry point for the flow from s to d (there could be none).

D. The LP for Scenario (c)

In this scenario, besides entry routelets there are also exit routelets for non-default paths. In addition to this, the default paths may not be used if an SD-pair has a valid entry routelet and a valid exit routelet, but the exit routelet in not on the default path.

We have the following difference in notation:

1) $f_{sd,m,k,o}$ is the flow from s to d redirected by the entry routelet on node m via relay routelet on node k towards the exit routelet on node o (the flow value is the variable we want to determine)
2) $P_{sd,k,m,o}$ is the path corresponding to the above flow (i.e. a known set of edges)
3) $\mathcal{B}_{f_{sd,m,k,o}}$ is a constant which denotes the capacity of the bottleneck link on the path $P_{sd,m,k,o}$.
4) $E_{sd,m}$ indicates whether the exit routelet on node m is on the default path from s to d.

The LP formulation for this case is as follows:

$$\min \quad \sum_{sd,m,k} f_{sd,m,k} + f_{sd} \tag{22}$$

$$\text{s.t.} \quad \sum_{k} M_k \leq B \quad \forall u \tag{23}$$

$$D_{sd,m} \leq M_m \quad \forall sd, m \tag{24}$$

$$\sum_{m} E_{sd,m} \leq 1 \quad \forall sd \tag{25}$$

$$\sum_{k} D_{sd,k} \leq \sum_{m} E_{sd,m} \quad \forall sd \tag{26}$$

$$f_{sd,m,k,o} \leq M_k \times \mathcal{B}_{f_{sd,m,k,o}} \quad \forall sd, k, m, o \tag{27}$$

$$f_{sd,m,k,o} \leq E_{sd,o} \times \mathcal{B}_{f_{sd,m,k,o}} \quad \forall sd, k, m, o \tag{28}$$

$$f_{sd,m,k,o} \leq D_{sd,m} \times \mathcal{B}_{f_{sd,m,k,o}} \; \forall sd, k, m, o \tag{29}$$

$$\sum_{e \in P_{sd,m,k,o}} f_{sd,m,k,o} \leq \rfloor_e \quad \forall e \in E \tag{30}$$

$$f_{sd,m,k,o}, M_k, D_{sd,m}, E_{sd,o} \geq 0$$

In the ILP version, variables $M_k, D_{sd,m}, E_{sd,o}$ take integer values.

We prove in the Appendix that the placement problems in Scenarios b and c are NP-hard.

IV. Rounding Algorithms

In this section we will provide a method to transform the fractional LP solution into an integer one. Our method is slightly different for the different scenarios. For scenario a, the algorithm presented offers provable guarantees, as stated in Theorem IV.1. For scenario b, the algorithm needs to be adapted, for reasons explained in section IV-B. While providing theoretical bounds for the performance of this modified algorithms remains an open problem, experimentally we observe that in scenario (b) we can match the optimal throughput, with no edge constraints violations, while using at most a $\log n$ factor more routelets.

A. Scenario (a)

Let $M_k, f_{s-d,m}, f_{sd}$ be the fractional values obtained after running a standard LP solver. Let $\tilde{M}_k, \tilde{f}_{s-d,m}, \tilde{f}_{sd}$ be the corresponding (integer) rounded values obtained as follows:

- Set $\tilde{M}_k = 1$ with probability M_k, otherwise set $\tilde{M}_k = 0$.
- If $M_k = 0$ set $\tilde{f}_{s-d,k} = 0$. [Note that if $M_k = 0$ then $f_{s-d,k}$ is also 0 due to constraint 7]
- If $M_k > 0$, set, for all SD-pairs $\tilde{f}_{s-d,k} = \frac{\tilde{M}_k f_{s-d,k}}{M_k}$.
- For all SD-pairs, let $\tilde{f}_{sd} = f_{sd}$.

Note that by construction $E[\tilde{M}_k] = M_k$.

In scenarios b and c we also have entry (and exit) routelet variables (such as $D_{s-d,m}$ and $E_{s-d,m}$). The rounding for these more complex scenarios requires further research.

Theorem IV.1 *The above rounding technique for scenario a produces a total throughput which matches the optimal throughput (in expectation). Furthermore, with high probability the constraints 6, 9 are not violated by more than a $\log K$ factor, where K is the total number of potential routelet locations. Constraints 7 and 8 are satisfied by construction.*

Proof:

We start by proving the first part of the theorem.

When $M_k > 0$, since $f_{s-d,k}, M_k$ are constants determined by the LP, and $E[\tilde{M}_k] = M_k$, we have that $E[\tilde{f}_{s-d,k}] = \frac{E[\tilde{M}_k] \times f_{s-d,k}}{M_k} = f_{s-d,k}$. $E[\tilde{f}_{s-d,k}] = f_{s-d,k}$ is true also when $M_k = 0$ (both flows equal 0 in that case). Thus, after rounding, by linearity of expectation we have that the rounded objective function (the total thorughput) matches the objective function of the LP (in expectation):

$$E\left[\sum_{sd,m} \tilde{f}_{sd,m} + \tilde{f}_{sd}\right] = \sum_{sd,m} f_{sd,m} + f_{sd}$$

Constraint 6 becomes, after the rounding, a sum of K Bernoulli random variables (since $\tilde{M}_k \in \{0,1\}$) with mean $\mu = E[\sum_k \tilde{M}_k] = \sum_k M_k \leq K$. We can apply a standard Chernoff bound [7] to obtain that with high probability (i.e. with probability at least $1/K$ (as long as $K, \mu \geq 3$) the sum will not exceed the expected value by more than a $\log K$ factor. The Chernoff bound is just the first inequality in the desired result:

$Pr\left[\sum_k \tilde{M}_k \geq (1 + \log K)K\right] \leq e^{\frac{(\log K)^2 \mu}{\log K + 2}} \leq \frac{1}{K}$

Finally, for every edge e, we want to bound the probability that the rounded flow on an edge (denoted by $\tilde{S}_e$ the summation term in (31)) exceeds the edge capacity by more than a $\log K$ factor, i.e. we want to bound:

$$Pr\left[\left(\sum_{sd,k} \mathcal{I}(e \in P_{sd,k}) \times \tilde{f}_{sd,k}\right) \geq (1 + \log K)\mathcal{C}_e\right] \tag{31}$$

For a fixed k such that $M_k \neq 0$ let $\tilde{F}_{k,e} = \sum sd\mathcal{I}(e \in P_{sd,k}) \times \tilde{f}_{sd,k}$ (and define $F_{k,e}$ similarly). These represent the sum of flows via relay k passing through edge e after the rounding (and before the rounding). Note that $\tilde{F}_{k,e} = F_{k,e} * \tilde{M}_k/M_k$, since $\tilde{M}_k/M_k$ is the transformation factor between the corresponding rounding and LP flow values.

After finding an LP solution, $F_{k,e} = \gamma_{k,e} \times \mathcal{C}_e$ for some constant $\gamma_{k,e} \leq 1$ (by constraint 8) and such that $\sum_{k:M_k \neq 0} \gamma_{k,e} \leq 1$ (by constraint 9).

Using this new notation, first note that $\tilde{S}_e = \sum_{k:M_k \neq 0} \tilde{F}_{k,e}$, which can be seen by summing first over k and then over sd in (31), and by noting that the contribution of flows for which $M_k = 0$ is zero both before and after the rounding.

Thus, bounding $\tilde{S}_e/\mathcal{C}_e$ is equivalent to bounding $\sum_{k:M_k \neq 0} \tilde{F}_{k,e}/\mathcal{C}_e = \sum_{k:M_k \neq 0} F_{k,e}/\mathcal{C}_{k,e} = \sum_k \gamma_{k,e} \times \tilde{M}_k/M_k = \sum_k \tilde{\gamma}_{k,e}$. Note that $\tilde{\gamma}_{k,e}$ is a random variable $\in [0,1]$ with $E[\tilde{\gamma}_{k,e}] = \gamma_{k,e}$. A generalized version of the Chernoff bound applies to the sum of these variables, with cummulative expected value equal to $\sum_{k:M_k \neq 0} \gamma_{k,e} \leq 1$, leading the desired result for $K > 5$: $Pr\left[\tilde{S}_e/\mathcal{C}_e \geq (1 + \log K)\right] < \frac{1}{K}$ ∎

In our simulations the capacity constraints bound is never violated by more than $\log K$ for $K > 5$.

B. Scenario (b)

The difficulty in selecting the routelet and determining the flow values based on the LP fractional solution in scenario (b) is due mainly to the fact that flows now depend on two routelets being present: the entry routelet as well as the relay (deflection) routelet corresponding to a flow.

If we restrict our attention to a single SD-pair, the cummulative LP value for the potential entry routelets is at most one. If we would round as in scenario (a), we could very well leave the SD-pair with no potential entry routelet. Since we would like to have a high probability (say $1 - 1/n$) that the SD-pair have an entry routelet after the rounding, we should round the variables with probability boosted by some factor. We choose the factor to be $\log n$, corresponding to the high probability $1 - 1/n$.

Topo	# nodes	Bandwidth distribution	B/w range (Mbps)	Node degree	# potential overlays	# SD pairs
0	50	HeavyTailed	10-1024	2	20	5
1	100	HeavyTailed	10-1024	2	40	20
2	200	HeavyTailed	10-1024	2	50	20
3	300	HeavyTailed	10-1024	2	100	30
4	400	HeavyTailed	10-1024	2	100	40
5	600	HeavyTailed	10-1024	2	100	50
6	100	Uniform	10-1024	2	40	20
7	100	Exponential	10-1024	2	40	20
8	100	HeavyTailed	50-5024	2	40	20
9	100	HeavyTailed	10-1024	5	40	20

TABLE II

TOPOLOGY CHARACTERISTICS (GENERATED USING BRITE)

Rounding the flows in this situation is another open problem, and our approach is to run the linear program with the choice of routelets now fixed (i.e. a max-flow problem) to obtain the maximum possible throughput that can be obtained given the random choice of routelets.

Experiments show that this approach leads to good results in practice.

V. EXPERIMENTAL METHODOLOGY AND EVALUATION

Our goal in this section is to comare through simulations on several BRITE topologies of varying node population, link, bandwidth and node degree. Table V shows the characteristics of the topologies we consider. We compare the performance of the following five algorithms:

LP: note the objective may be higher than achievable with integer constraints;

ILP: where possible; may be NP-hard to solve in general;

Rounding: using the different rounding schemes for scenario a and scenario b presented in Section **??**;

Greedy: In scenario b, first pick entry routelets for all sd-pairs (first potential routelet encountered is designated the entry routelet for a given sd-pair). Step 2 (the only step for scenario a): compute the sum of bottleneck values for the paths introduced by the remaining routelets. Select the routelet with the highest bottleneck. Repeat step 2 until the desired number of routelets is selected;

Random: First step: pick the desired number of routelets randomly. For scenario b, for each SD-pair also pick an entry points randomly (if available). Since the entry point is subject to constraint 15) many SD-pairs do not have an available routelet to select as entry point after the first step.

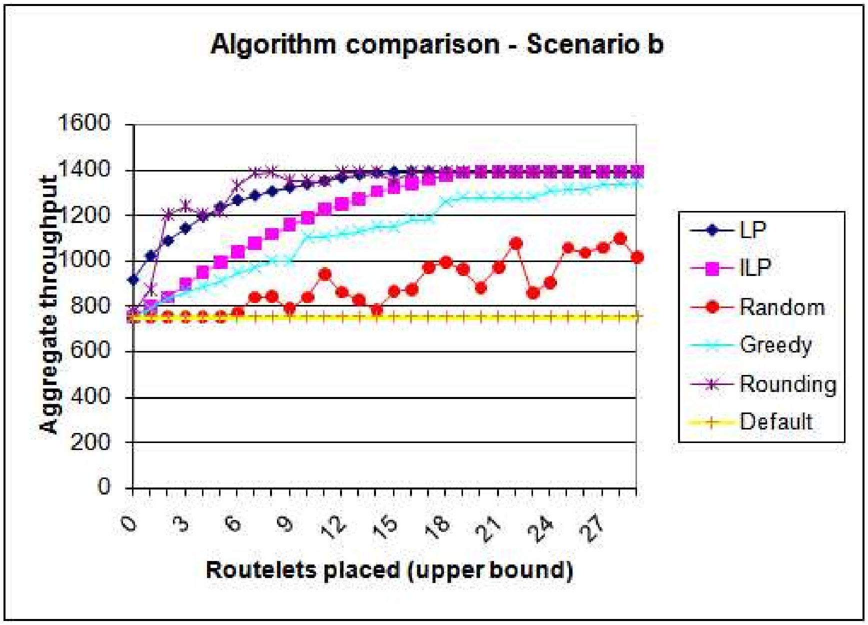


Fig. 2. Aggregate throughput as the total number of routelets is increased for the five algorithms compared. Scenario b, Topo1.

In order to construct the linear program common information regarding the topology, such as the nodes, the edges, the bandwidth on the edges, and also how the nodes are connected to each other needs to be gathered, besides the information regarding the SD-pairs, K, and eligible transit node locations.

In practice, tools such as *traceroute* and *pathchar* can be used to obtain the connectivity information, underlay routing, and bottleneck bandwidth. In our experiments, we use ns to collect this information.

We next present the results for the scenario b, followed by the scenario a results.

A. Results - Scenario b

Experiments show that the rounding algorithm achieves the maximum throughput while using no more than $\log r$ extra routelets.

Figure 2 illustrates the behaviour of our five algorithms as we increase the bound K on the number of routelets we can place. The tests are run on Topo1. Note how close the rounding output is to the LP output, exceeding the feasible ILP value. This is due in part to the fact that the rounding may exceed the number of routelet (by no more than $\log K$), allowing the rounding to discover the optimum value before the LP does (at $K = 9$). K is an upper bound (an approximative upper bound for the rounding algorithm), and once the LP/ILP/rounding reach a stable optimum solution, around $K = 20$, no more than this number of routelets is used. The performance of random is an average over 4 runs. The greedy solution is surprisingly good at extracting a high percentage of the flow achieved by the LP/ILP solutions. For comparison, we include the aggregate throughput obtained with default single-path routing.

Figure 3 captures the throughput per SD-pair for Topology 1, with $K = 20$ for the 16 SD-pairs with non-zero throughput for one or more akgorithm. Note the default bar, which, when not present indicates zero default flow between the specific SD-pair. By careful placement of routelets, LP, ILP and the rounding algorithms are able to maximize the throughput achieved for each SD-pair. Random misses some key entry routelets and thus is often unable to improve the throughput. Greedy correctly selects the entry routelets, but does initially miss some less significant deflection routelets which participate in the optimum solution.

In Figure 4 we study the aggregate throughput for Topology 1, as the number of SD-pairs we want to create multiple path for increases. The target total number of routelets K is 8. Rounding is outperforming the LP/ILP solutions since it is allowed to use a few more routelets than the LP/ILP solutions.

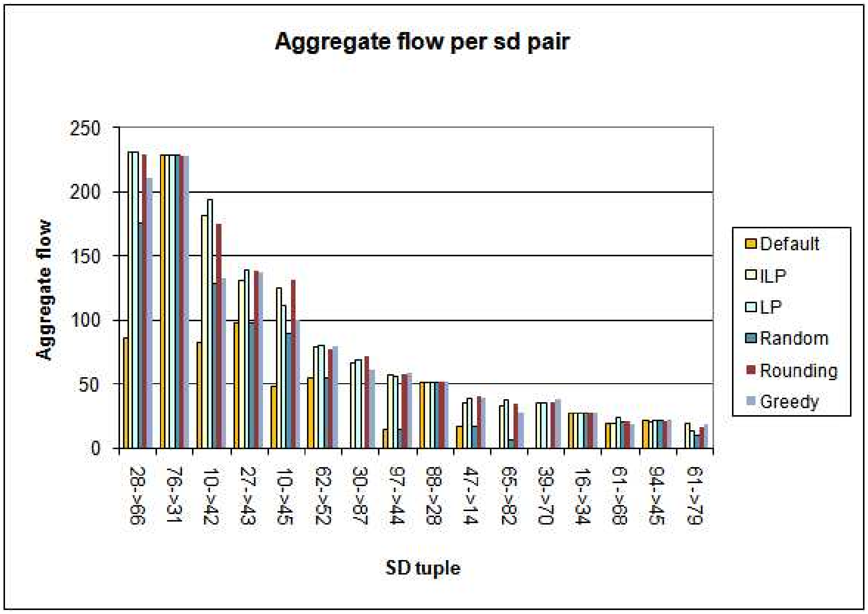


Fig. 3. Throughput per sd-pair for the five algorithms. Scenario b, Topo 1, target number of routelets = 20.

We plot the number of routelets placed by the rounding algorithm for this same example in Figure 5. The more SD-pairs, the more complex the solution is. As the complexity increases, more nodes are usually necessary to achieve a better solution. While the LP uses fractional values for the routelets to achieve the maximum throughput, the rounding can either select or totally discharge a potential routelet. By referring to Figure 4 we note that the range where the rounding algorithm selects more than twice the number of routelets of the LP, corresponds to the range where the rounding algorithm outperforms the LP solution. The total number of routelets used remains below the theoretical bound of $K \log K = 24$.

In Figure 6 we present the performance of our various algorithms across the ten topologies. The aggregate throughput values are normalized by dividing the throughput of each algorithm by the LP throughput. The actual LP throughput for each topology is listed in parentheses (in the x axis names). Using the rounding technique a multipath protocol can achieve a 2-5 fold increase in bandwidth utilization over the default single path routing.

B. Results - Scenario a

Recall that the rounding algorithm is different for the two different scenario, thus the results in these section are slightly different than those in Section V-A.

In Figure 7 the rounding throughput is computed as the average over 4 distinct trials. The results and conclusions are the similar to the ones for Figure 2. The main difference is a higher variability in the rounding results compared to scenario (b) due to the absence of the boosting factor present for scenario (b).

In Figure 8 note that the rounding throughput can significantly exceed the LP value for a given SD-pair. More than 30% higher for pair 76-31 for example. This is different from scenario (b) and indicates a probable edge violation. It would be interesting to plot the average behaviour as opposed to one individual rounding example.

The results in Figure 9 are similar to the scenario (b) Figure 4.

We obtain different results in Figure 10 than in Figure 5. The number of routelets selected matches much better the desired number of routlets in scenario (a) than in scenario (b).

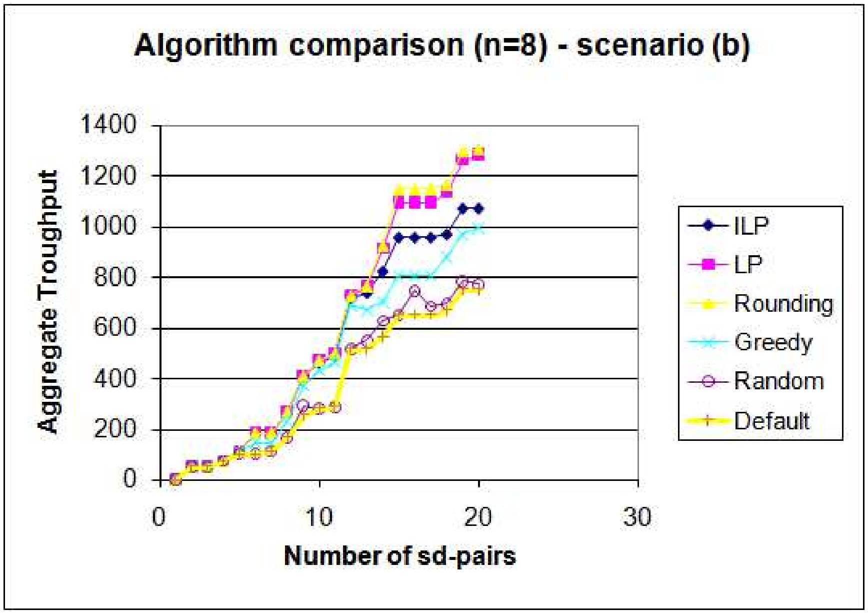


Fig. 4. Aggregate throughput as the total number of SD-pairs is increased. Bound on the number of routelets is 8. Scenario b, Topo1.

Figure 11 illustrates the edge-violations for scenario (a) rounding. Note there is no bound for $r < 3$ and in this regime we observe the largest values of egde-violation. For the remaining part of the range, the maximum edge-violation remains under the theoretical bound.

Finally Figure 12 illustrated the performance accross the various topologies we tested. Similar to Figure 6.

VI. Related Work

Multipath protocols have been well researched since Maxemchuk's seminal work on dispersity routing [20] to improve throughput and resilience to path failures or packet losses [27], [8]. Striping or inverse multiplexing [3], [9], [24], [17] provides link level mechanisms for splitting input flows among multiple links to increase throughput. More recently, multipath techniques have been proposed in the context of overlay routing or multihomed clients [5], [2], [28]. Katabi et al. [14] and Elwalid et al. [10], propose splitting aggregate traffic flows along multiple paths to achieve load balancing and stability in the context of intradomain traffic engineering. Harp [18] uses multiple paths for scheduling transfers with heterogeneous requirements. Multipath TCP [11] and KV [15] are based on a utility-theoretic framework for network-wide resource allocation, and systematically address fairness. These controllers maintain a nontrivial sending rate on each available path so as to maximize utilization and global fairness.

A key requirement of all these works is the support for letting end-hosts *utilize* multiple paths in the Internet for transmitting packets. This can be achieved by employing "packet relayers" that are realizable in several ways. A handful of works address a similar node placement problem in the context of providing resilience to path failures between a source and a destination [12], [29], [25], and for improving end-throughput by breaking an end-to-end TCP connection into a series of shorter-loop connections [19], [25]. However, to the best of our knowledge, ours is the first work to address the placement problem in the context of maximizing network utilization.

Node placement has also been studied in other contexts such as Web server and cache placement [13], [23]. Qiu et al. [23] study a web server replica placement problem to minimize the cost for clients to access data. Jamin et al. [13] study mirror placement problem, in which mirrors can be placed only at

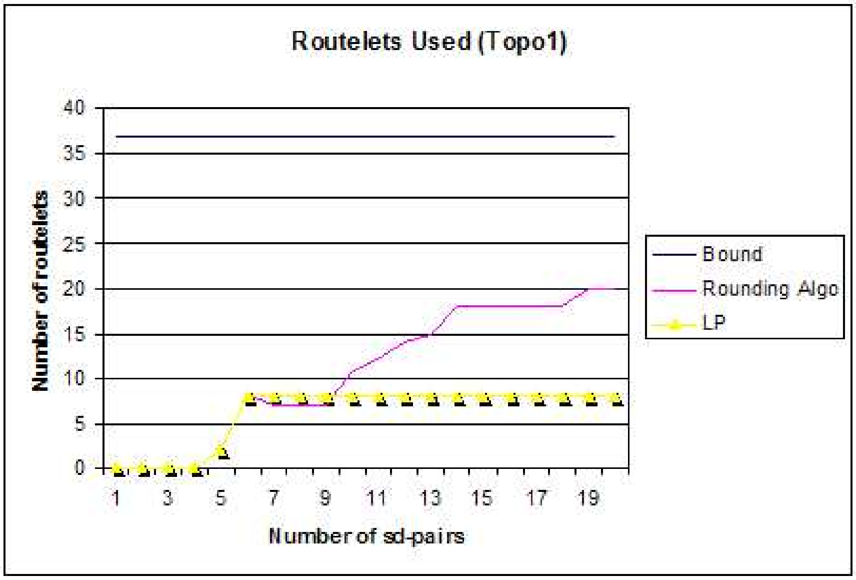


Fig. 5. Number of routelets used as we vary the complexity of the problem (sd-pairs). Scenario b, Topo1, target number of routlets = 8.

a restricted set of locations, much like our problem constraint. Also, they show a diminishing return of placing more and more mirrors, similar to our observations.

As also argued by Key et al [16], we believe that multipath routing forms a powerful building block for a Robust Internet Architecture of the future. In this paper, we discuss three scenarios in which such an architecture may be realized.

VII. Conclusion

In this paper, we address the routelet placement problem to assist multipath transport protocols that are designed to achieve better network utilization and fairness. We identify three different deployment scenarios of routelet placement, provide LP formulations for placement in each of the scenarios, prove hardness results. We provide rounding algorithms with provable properties for a subset of the scenarios, and compare their performance through simulations on several BRITE topologies of varying scales.

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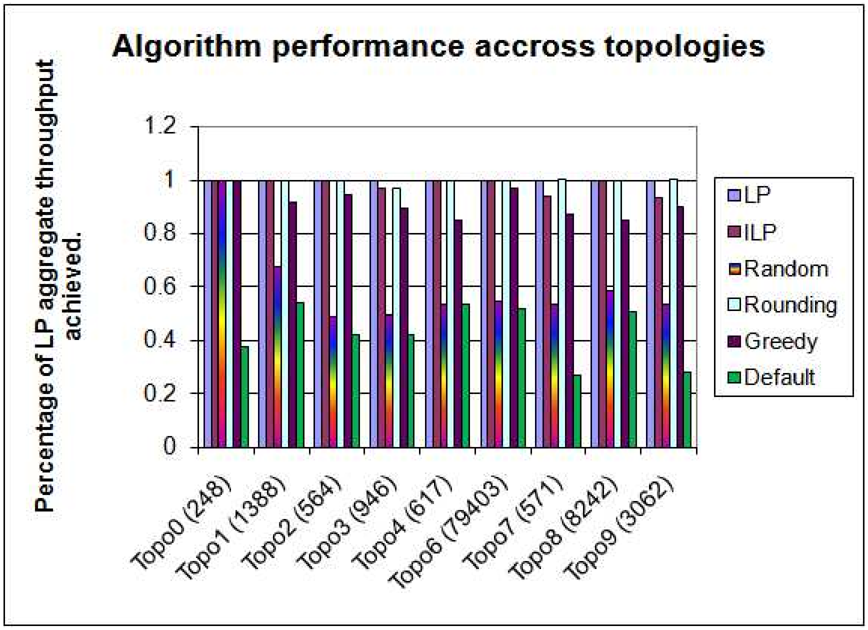

Fig. 6. Performance accross various topologies for scenario b. Target number of routelets = 20.

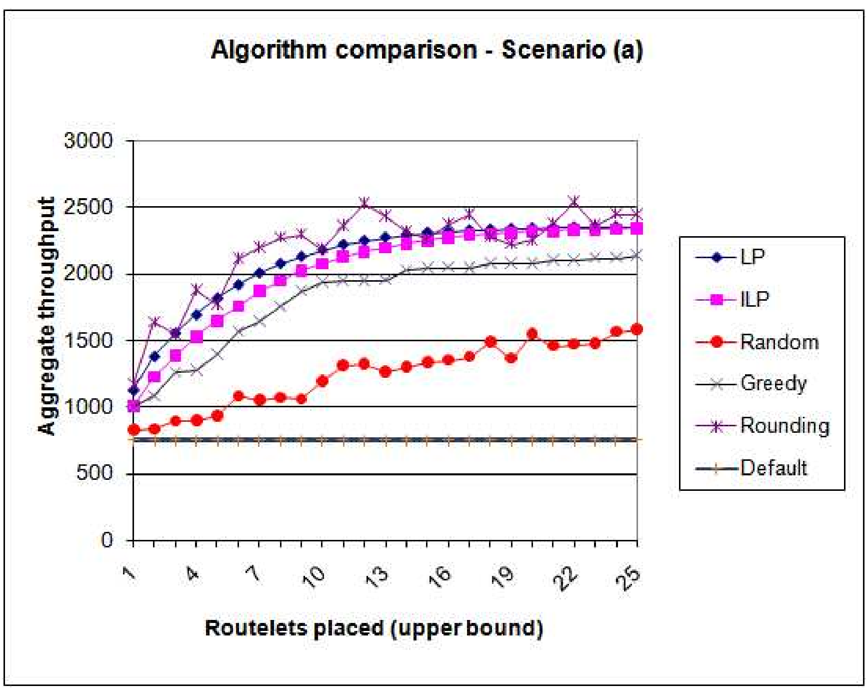

Fig. 7. Aggregate throughput as the total number of routelets is increased for the five algorithms compared. Scenario a, Topo1.

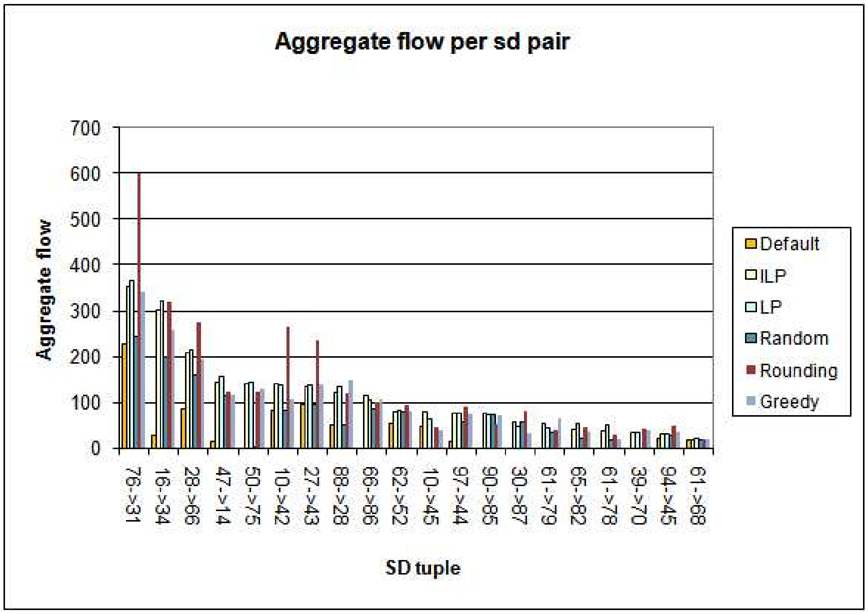

Fig. 8. Throughput per sd-pair for the five algorithms. Scenario a, Topo 1, target number of routelets = 20.

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APPENDIX

The problem of placing a given number of relay nodes in an optimum manner for maximizing network utilization is at least NP-hard for Scenarios b and c. We will explain in detail a reduction from Balanced

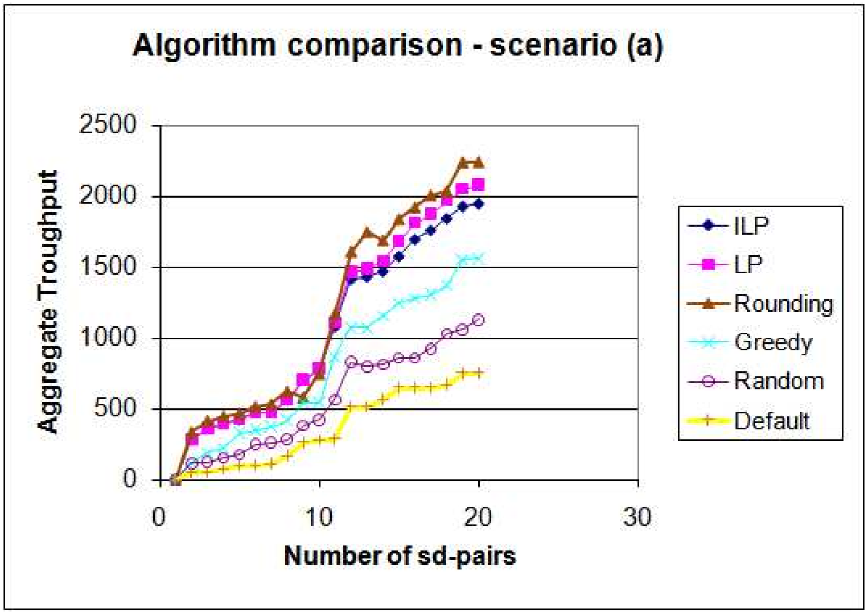

Fig. 9. Aggregate throughput as the total number of SD-pairs is increased. Bound on the number of routelets is 8. Scenario a, Topo1.

Complete Bipartite Subgraph (BCBS) to Scenario (b).

BCBS is the problem of deciding, given a bipartite graph G with vertex set $S \cup T$, edges (s, t) such that $s \in S, t \in T$ and a parameter b whether there exist a subset of nodes $S\prime \subset S$ and $T\prime \subset T$ such that $|S\prime| = |T\prime| = b$ and all such edges are edges in G.

We reduce this problem to the decision version of our problem, which is: *Given a graph G that represents the routers and links in the Internet, a subset of the routers that are capable of providing the relay service, and a set of SD-pairs, can we place at most k relay services as to increase the bandwidth utilization between the SD-pairs by an amount equal to a (assuming the paths an SD-pair can utilize to send info is either a default path or a S-E-R-D path as in Scenario b)?*

We obtain the reduction from BCBS to the above problem as follows:

Create the network by copying G and adding a destination node D. We connect all nodes in S with the destination (the dotted lines in Figure 13) and also all nodes in T are connected with the destination (the plain lines in Figure 13). Every node in S is a source, and the direct links represent the default SD paths. The cloud represents the edges from G. All links have capacity 1. Routelets can be placed at either S or T. We ask: is it possible to place $2b$ nodes and get a b^2 increase in the bandwidth (so $k = 2b$ and $a = b^2$)?

Now we will show that solving the node placement question is equivalent to solving the BCBS problem. Any node assignment that distributes the nodes between the two sides, say r nodes in S and l nodes in T, can add at most rl capacity in terms of bandwidth. This product is maximized when $r = l = k/2 = b$, since the sum $r + l = k$ is constant, and the maximum value is b^2. No unbalanced distribution can lead to an extra capacity of b^2 or more (because the product rl is strictly less than that in the unbalanced case). The balanced distribution only leads to an extra capacity of b^2 if the nodes selected form a complete bipartite clique. Thus, to be able to detect whether nodes can be placed to achieve this improvement in bandwidth or not is to be able to solve the BCBS problem.

Since solving the BCBS problem is NP-hard, determining the answer to the decision version of Problem b is also NP-hard.

The reduction to Scenario (c) is similar, the only difference is that in scenario (c) we need an exit

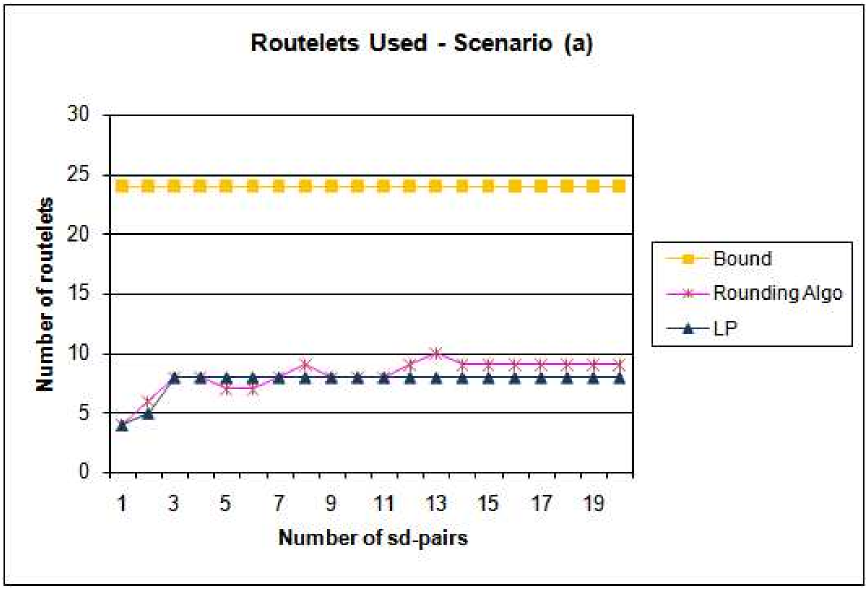

Fig. 10. Number of routelets used as we vary the complexity of the problem (sd-pairs). Scenario a, Topo1, target number of routlets = 8.

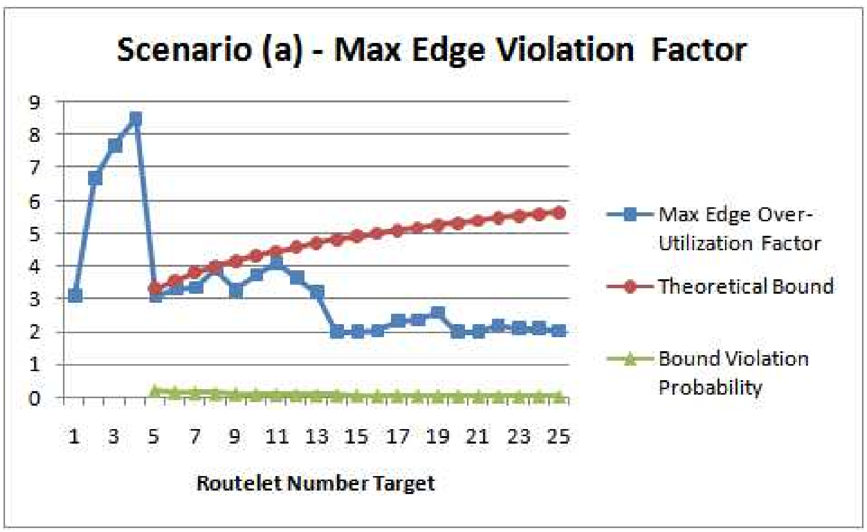

Fig. 11. Actual max-edge violation factor as we vary the target number of routelets versus the theoretical upper bound (available for $r > 3$ only).

routelet on the default path. Since the default paths are a single hop, that routelet needs to be placed at the destination node D in order to allow more flow than the default. Placing the exit routelet at the source node creates no additional paths beyond the already existing default path.

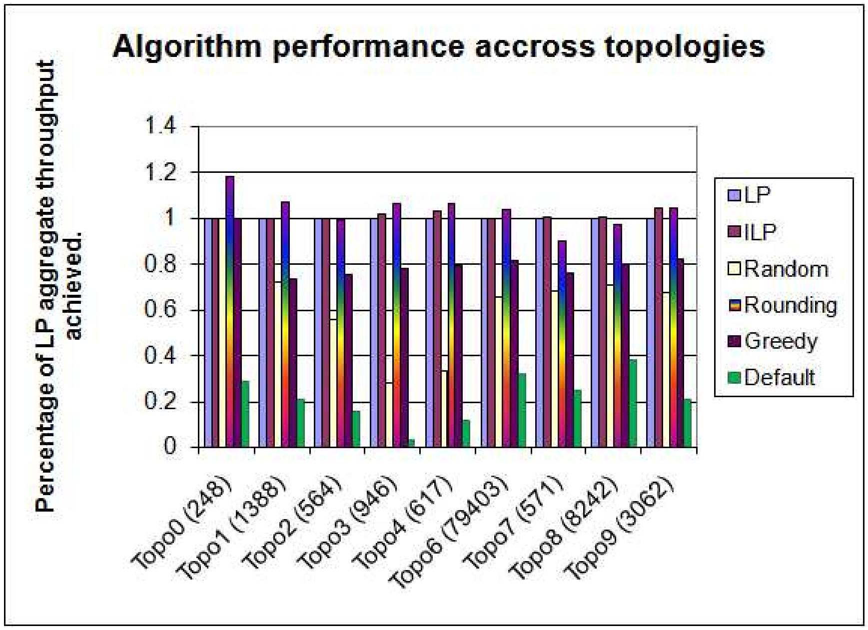

Fig. 12. Performance accross various topologies for scenario a. Target number of routelets = 20.

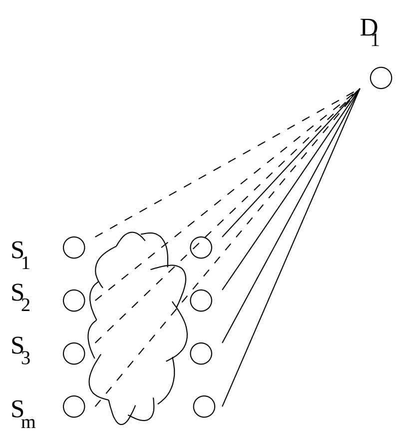

Fig. 13. Reduction from Balanced Complete Bipartite Subgraph