The Cause of Brownian Motion

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Abstract

For the past 200 years, the Brownian motion of small particles has been observed and studied due to its significance in understanding physics at the microscopic scale. Albert Einstein statistically modeled the behavior of these particles, but the mechanism driving Brownian motion has remained unclear. It was previously suggested that molecules push the particles in Brownian motion. However, this study demonstrates that the random movement of molecules cannot provide sufficient momentum to drive Brownian motion. Instead, we propose that Brownian motion is driven by transimpact—an interaction between atoms and a side effect of atomic electron transitions. When energy is absorbed, an electron is excited to a higher orbital, increasing the volume of the atom and reducing the distance to adjacent particles. This disrupts the existing spatial arrangement between atoms and alters the balance of attractive and repulsive forces. The transition induces a repulsive force between particles, with an explosive impact that pushes the particles apart. Transimpact may indeed be the fundamental interaction behind many physical processes and phenomena, such as particle motion, phase transitions, and the rotation of Crookes radiometers. Additionally, transimpact facilitates the exchange of internal energy between a system's potential and kinetic energy, as well as among different components, thereby maintaining equilibrium.

Introduction

In 1827, while studying the pollen grains of the plant Clarkia pulchella suspended in water under a microscope, Robert Brown observed the jittery motion of minute particles.[1-5] By repeating the experiment with particles of inorganic matter, he ruled out the possibility that the motion was life-related. This motion was later named after him as Brownian motion. Since then, it has been recognized that Brownian motion is not unique to pollen but is a common phenomenon observable in daily life. Most people might have noticed dust particles dancing in a beam of light in a dark room. Indeed, Brownian motion reflects a fundamental interaction between particles at a microscopic scale in physics.

In 1905, Albert Einstein statistically modeled the movement of particles in Brownian motion,[6-8] providing a method to indirectly demonstrate the existence of atoms and molecules, which were not well established at the time. While it was postulated that the random movement of fluid molecules causes the particles to move in Brownian motion, the underlying physics governing this motion remains unclear to this day.

Can the randomly moving molecules have sufficient momentum to drive the particles in Brownian motion? If not, what force is responsible for pushing the particles? These questions may relate to some of the fundamental processes underlying thermodynamics and many other physical phenomena, which is the primary motivation of this study.

Insufficient Momentum from Molecule Movement

To estimate the velocity required for a molecule to drive Brownian motion, consider a grain of pollen initially at rest, struck by a single water molecule traveling at velocity v. To maximize the estimated momentum transfer, we assume an idealized scenario in which the water molecule rebounds with the same speed in the opposite direction, resulting in a complete momentum transfer to the pollen. The mass of a water molecule is approximately 2.99 x 10^{-26} kg. Under this assumption, the change in momentum of the water molecule during the collision is v x 2 x 2.99 x 10^{-26} kg•m/s, or

$$(1) \qquad P = v \times 5.98 \times 10^{-26}\, kg\bullet m/s$$

The pollen grain of *Clarkia pulchella* typically measures between 50 - 100 x 10^{-6} m in diameter. For a conservative estimate, we use a smaller diameter of 40 x 10^{-6} m, corresponding to a volume of 6.4 x 10^{-14} m^3. Since the pollen is suspended in water, its density can reasonably be assumed to be close to that of water, about 1000 kg/m^3 at room temperature. Suppose the pollen acquires a velocity of 300 x 10^{-9} m/s, a speed that is barely detectable by the naked eye under a microscope, as the distance moved in a second is already less than the wavelength range of visible light (380-750 x 10^{-9} m). In other words, the actual speed of pollen observed by Brown was likely greater than this threshold. Following a collision, the pollen would gain a momentum of approximately 1.92 x 10^{-18} kg•m/s. This momentum should correspond to the change in momentum imparted by the colliding water molecule:

$$(2) \qquad v \times 5.98 \times 10^{-26} = 1.92 \times 10^{-18}$$

So, to push the pollen at the minimum detectable velocity of 300 x 10^{-9} m/s, the speed of a water molecule must be greater than:

$$(3) \qquad v = 3.21 \times 10^{7}\ m/s$$

This velocity is greater than 10% of the speed of light. At room temperature, the average velocity of water molecules, derived from the kinetic energy distribution, is approximately 590 m/s.[9] This value is consistent with the speed of sound in water (~1,500 m/s),[10] which can be considered an upper bound for molecular velocity, as sound propagation depends on molecular interactions. For comparison, the average velocity of air molecules is less than 1,200 m/s. Given that water is significantly more viscous than air, the motion of water molecules is even more restricted. The required speed in this scenario is four orders of magnitude greater than the actual molecular speed. Even under this highly conservative estimate, the speed required for a water molecule is far beyond what is physically achievable. Therefore, the velocity needed to explain Brownian motion through direct molecular impacts is physically implausible.

Furthermore, let us estimate the probability that a particular water molecule could exceed a speed of 3.21 x 10^{7} m/s. Assume the speed of water molecules follows a Gaussian distribution with a mean of 1,500 m/s, which, based on the previous discussion, represents the upper bound of their typical velocity of random motion. To maximize the estimated probability, we adopt a highly generous assumption: a standard deviation of 1,500 m/s, equal to the mean. This is hundreds of times broader than the actual spread in a realistic Maxwell–Boltzmann distribution for water molecules. Under this assumption, the z-score corresponding to a speed of 3.21 x 10^{7} m/s is:

(4) $$z = \frac{x - \mu}{\sigma} = \frac{3.21 \times 10^{7} - 1{,}500}{1{,}500} = 21,399$$

Therefore, the probability of a water molecule reaching a speed greater than 3.21 x 10^7 m/s, based on a Gaussian distribution, can be estimated as follows:

(5) $$P\left(v > 3.21 \times 10^{7}\right) = P(z > 21,339) \approx \frac{1}{z\sqrt{\pi/2}} e^{-z^{2}/2} < 10^{-98{,}000{,}000}$$

This probability is effectively zero. Even if we assume that the pollen is struck by surrounding water molecules every yoctosecond, the expected time for a molecule to reach such an extreme velocity would vastly exceed the age of the universe. Consequently, the velocity required of water molecules to account for the observed Brownian motion lies well beyond the limits of physical plausibility.

Nevertheless, one might argue that the simultaneous impact of multiple water molecules could collectively impart sufficient momentum to move the pollen. However, considering the average velocity of water molecules (~590 m/s), accelerating a typical pollen grain to a minimum detectable speed of 300 x 10^{-9} m/s would require the simultaneous collision of more than 54,418 molecules in the same direction. This number, denoted as N, can be estimated using the principle of momentum conservation, as expressed by the following equation:

(6) $$N \times 590 \times 5.98 \times 10^{-26} = 1.92 \times 10^{-18}$$

Given the inherently random nature of molecular motion, water molecules collide with the pollen from all directions, causing their effects to largely cancel out. To produce a net displacement of a single grain of pollen, all 54,418 molecules would need to strike one side simultaneously, with no opposing collisions from the other side. However, in three-dimensional space, each molecule can travel in an infinite number of directions, making such coordinated alignment extremely improbable. The chance that all 54,418 molecules move in nearly the same direction at the same time is no greater than the probability estimated in Equation 5. Even under a highly simplified assumption, where each molecule has only two possible moves: hit or not hit, the probability of this synchronized action is already vanishingly small:

(7) $$P = 1 \times 2^{-54{,}418}$$

The likelihood of such a highly coordinated and asymmetric impact occurring spontaneously is, for all practical purposes, negligible. Consequently, the hypothesis that Brownian motion results from the collective effect of random molecular collisions lacks sufficient support. If the motion of molecules cannot provide the necessary momentum to drive Brownian motion, then what mechanism is truly responsible?

The Mechanism Driving Brownian Motion

"Transimpact", a term derived from "transition impact", refers to an interaction between atoms resulting from a side effect of atomic electron transitions.[11] Typically, attractive forces are induced between atoms or molecules that are close. When the space between them becomes too small, the repulsion of their valence electrons increases rapidly. At a certain distance, a balance is achieved between these repulsive and attractive forces. This balance is often disrupted by atomic electron transitions. When an electron absorbs energy, such as from electromagnetic waves, it will be excited to a higher orbital, usually within a few nanoseconds.[12-13] As a result, the electron cloud and the atom's volume expand, reducing the distance to adjacent atoms. This sudden reduction in space disrupts their balance and significantly increases the repulsion between them. The impact is explosive and can exert a substantial force on adjacent particles.

As illustrated in Figure 1A, two single-atom molecules are positioned close to each other, with their attraction and repulsion balanced at a small separation. Upon absorbing energy, an electron in one of the molecules undergoes an atomic orbital transition, increasing the molecule's size and reducing the space between them, as shown in Figure 1B. This expansion increases the repulsion between the molecules, pushing them apart and causing them to move in opposite directions, as depicted in Figure 1C.[11,14]

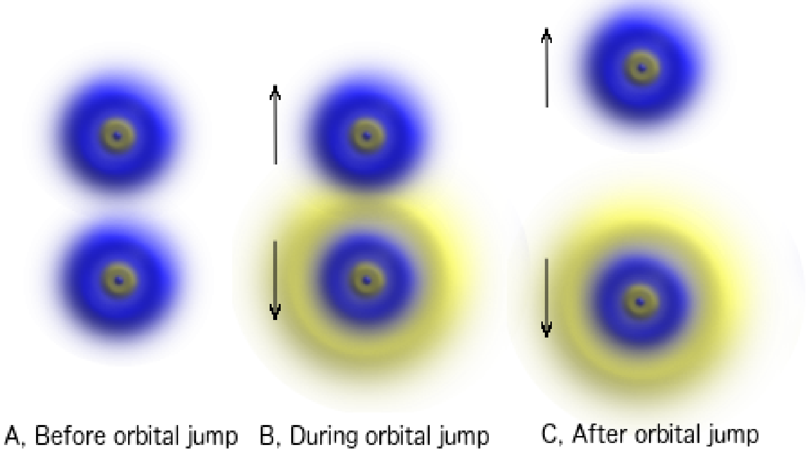

Figure 1: Process of transimpact.

To illustrate this interaction, let us consider the electron orbital transition of an oxygen atom. An atomic radius is primarily determined by its valence orbital. An oxygen atom has 8 electrons, with a ground state configuration of $1s^2 2s^2 2p^4$, giving it a radius of approximately 48 pm. In its typical excited state, the configuration is $1s^2 2s^2 2p^3 3s^1$, with a radius around 118 pm. Thus, when an oxygen atom becomes excited, its radius more than doubles, leading to a volume increase of over 1,200% within a few nanoseconds. This expansion effect is likely even more pronounced in hydrogen atoms. This rough estimate demonstrates how a water molecule could expand significantly as it absorbs energy from its surroundings.

The impact of transimpact can be visualized using the burst of popcorn. Consider the momentum exerted on adjacent objects by the popcorn as it pops. This analogy is similar to the interaction that occurs when an electron in a water molecule becomes excited next to a pollen grain. It is believed that this mechanism drives the Brownian motion of pollen in water.

Since atomic electron transitions are common interactions at the microscopic scale, transimpact is likely a fundamental mechanism, not only responsible for Brownian motion but also influential in a wide range of physical processes. The particles or objects affected by transimpacts may include atoms, molecules, pollen grains, dust particles, or even the walls of a gas container. When a transimpact occurs between two molecules, they are pushed apart and gain kinetic energy, which manifests as vibrational motion in solids or random motion in fluids. In essence, transimpacts facilitate the conversion of radiative and potential energy into kinetic energy. That is, kinetic energy can be understood as emerging from radiative or potential sources through transimpact events. Several thermodynamic processes involving transimpacts are examined in the following sections.

The Force Driving Crookes Radiometers

Transimpact offers a compelling explanation for the operation of Crookes radiometers.[14] These devices feature vanes that rotate when exposed to light, a phenomenon that, despite being observed for over a century, remained poorly understood. Numerous theories have been proposed, yet none have fully accounted for all experimental observations, particularly the initial maximum acceleration of the vanes. According to the transimpact theory, when an electron transition induces a transimpact between an air molecule and the vane surface, momentum is transferred to the vane, causing it to move. Under illumination, the blackened side of each vane absorbs more energy than the reflective white side, leading to a greater frequency of transimpacts on the black side. This asymmetry in impact frequency generates a net torque, resulting in the observed rotation. Experimental data from light exposure studies provide strong support for this mechanism.

Most existing theories attribute the rotation of the vanes to airflow or differences in air pressure inside the radiometer. For these theories to be valid, the air within the radiometer would need to be heated to create pressure differentials, a process that takes time. For the vanes to start rotating, the driving force must be strong enough to overcome the friction between the vanes and the mounting spindle. Consequently, the vanes cannot move initially until this friction is overcome. Furthermore, even after rotation begins, acceleration should start from zero and gradually increase, as it is proportional to the net driving force and should rise as the air heats up. However, this initial phase of zero or low acceleration is absent in nearly all observations.

Time (s)	0	2	5	17	30	43	64	78	100	110	120	128
Speed (Hz, rev/s)	0	7.5	15	22.5	25	22.5	15	7.5	3.75	3	3	0
Acceleration (Hz/s)		3.75	2.5	0.625	0.192	-0.192	-0.357	-0.536	-0.17	-0.075	0	-0.375

Table 1: Changes in radiometer speed and acceleration during the flashlight experiments.

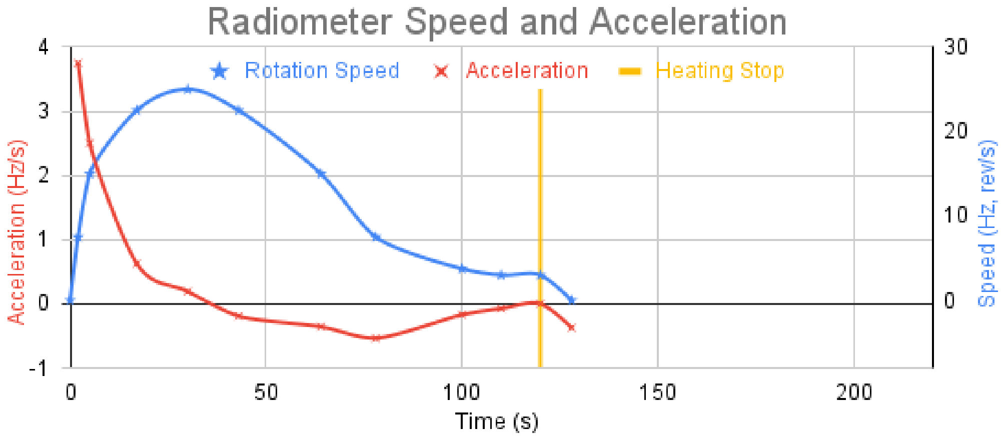

Figure 2: Changes in radiometer speed and acceleration during the flashlight experiment.

In contrast, the transimpact theory predicts that acceleration begins at its maximum because atomic electron transitions occur as soon as the radiometers are exposed to light. As the rotation gains speed, air resistance increases, which reduces the driving force and, consequently, the acceleration. Therefore, the rotation speed should initially increase until it reaches a peak, where the driving force is balanced by air resistance. This pattern is observed in almost all experiments. To further verify this prediction, an experiment was conducted using a powerful flashlight. The speed changes observed during the experiment are compiled in Table 1. Each acceleration value in the table represents the average between two sample points of the rotation speed.

The experimental data presented in Table 1 are plotted in Figure 2. The striking agreement between the observations and predictions provides compelling evidence for the validity of the transimpact theory in explaining the behavior of Crookes radiometers.

Misconception about Temperature Representation

It is commonly believed that temperature represents the average kinetic energy of particle motion in substances. This misconception will be clarified by examining the role of transimpacts in particle interactions. Transimpacts cause particle vibrations in solids and motion in fluids. In gases, this motion becomes so pronounced that it leads to the belief that the kinetic energy of particle motion is the primary component of thermal energy, which temperature represents. This misconception is further reinforced by the empirical law of ideal gases. Based on this law, the kinetic theory of gases was developed, establishing a correlation between temperature and the average speed of gas molecules:

(8) $$v = \sqrt{\frac{3k_bT}{m}}$$

Here, m denotes the mass of a gas molecule, T indicates the absolute temperature, k_b represents the Boltzmann constant (1.38×10^{-23} J/K), and v is the predicted average speed of the gas molecules.[15-21] Therefore, the average kinetic energy due to the motion of molecules in an ideal gas can be derived from the kinetic theory:

(9) $$K = \frac{1}{2}mv^2 = \frac{3}{2}k_bT$$

This suggests the kinetic energy K in the system is proportional to the absolute temperature of the system. This equation reflects the common belief that temperature represents the average kinetic energy in a system. If kinetic energy were the sole determinant of a system's temperature, changes in temperature and kinetic energy could be used to calculate specific heat. Consequently, the specific heat of ideal gases can be predicted using the kinetic theory of gases:

(10) $$C_k = \frac{K}{mT} = \frac{3k_b}{2m}$$

This corresponds to a prediction of isometric molar heat capacity:

(11) $$C_v = \frac{3R}{2}$$

Here, R denotes the ideal gas constant (8.31 J/mol•K). This prediction closely matches measurements for gases with monatomic molecules with an accuracy of 99% or higher. However, the kinetic theory of gases fails to accurately predict the specific heat for gases with polyatomic molecules. The predicted values for gases with polyatomic molecules are consistently lower than the measured values. The prediction error appears to be roughly proportional to the number of bonds in the gas molecules, as illustrated in Figure 3.[22]

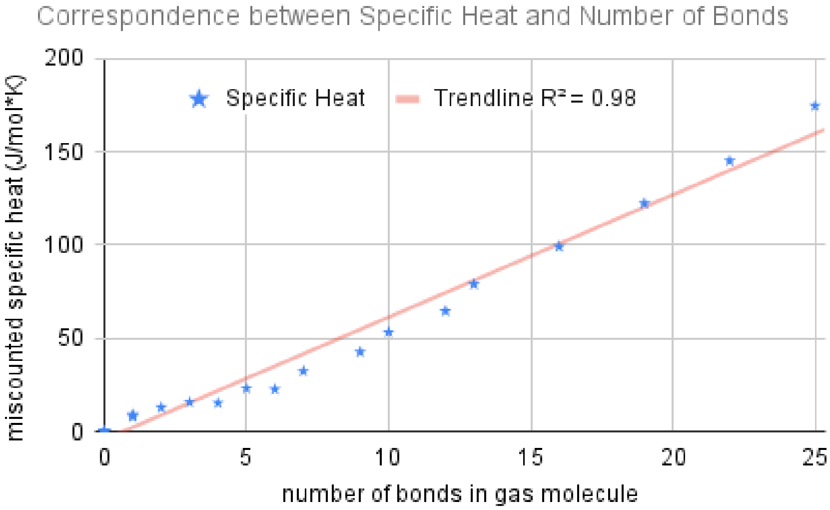

Figure 3: The correspondence between unaccounted molar-specific heat in the kinetic theory and the number of bonds. The unaccounted molar-specific heat is measured by offsetting the molar-specific heat predicted by the kinetic theory. The slope of the fitted trendline is about 6.56 with an R^2 of 0.98.

For instance, the predicted specific heat for octane is only 7% of the measured value, indicating a 93% prediction error. In other words, only 7% of the specific heat energy is allocated to increasing the kinetic energy for molecular motion. Numerous adjustments have been introduced to the theory, such as the equipartition of kinetic energy among vibrational, rotational, and translational motions, and considerations of the degrees of freedom in complex molecules.[23-30] However, these adjustments have all failed to account for the specific heat discrepancies shown in Figure 3.

The root cause of this discrepancy lies in the misconception that kinetic energy solely represents temperature. In reality, temperature measures thermal energy, which encompasses both kinetic and potential energy. Potential energy can be stored between particles under the influence of certain forces, such as chemical bonds. Bonds act like springs for energy storage: when work, such as specific heat, is done to expand a bond's space, the additional energy is stored in the extended bond. Similarly, energy can be stored in an excited electron in higher orbitals. When the electron returns to a lower orbital or the bond contracts, the stored energy is released as electromagnetic waves, which may be absorbed by other electrons or bonds. Thermal energy is the dynamic portion of internal energy and is typically measured by thermometers as temperature. This explains the correlation between the mispredicted specific heat and the number of bonds in gas molecules, as shown in Figure 3.

A transimpact occurs in every potential energy exchange process between different components, where a fraction of the potential energy is converted into kinetic energy for particle motion. Thus, kinetic energy is derived from potential energy and mirrors its changes, creating the delusion of a direct correlation with temperature. However, this relationship is indirect, mediated by potential energy, and will break down during a phase transition at a constant temperature, as discussed next. Consequently, temperature is primarily influenced by potential energy. This becomes clearer when considering the nature of absolute zero temperature, as kinetic energy is not zero even at absolute zero.[31]

The Cause of Phase Transition

Transimpact may also be the true cause of phase transitions.[32] A common textbook misconception attributes phase transitions to the increasing intensity of molecular vibrations at high temperatures. Phase transitions are influenced by the interplay of three forces within a system: pressure and bond forces that hold molecules together, and transimpacts that tend to break them apart. The balance among these three forces determines the state of matter. For instance, molecules in a solid can be held together because

$$(12) \quad T < B + P$$

Here, T denotes the average transimpact force, B indicates the bond force, and P stands for the pressure. A phase transition occurs when

$$(13) \quad T > B + P$$

The impact of transimpacts is more intense than that of molecular vibrations, as kinetic energy is derived from transimpacts. Bonds are likely to be broken by transimpacts before vibrations reach such high intensity. This theory also provides a better explanation for phase transitions occurring at constant temperatures.

This relates to another common textbook misconception, which posits that since the energy in a phase transition is used to break bonds between molecules, there's no net increase in the particles' kinetic energy, and therefore, no rise in temperature. An implicit assumption in this notion is that temperature represents the average kinetic energy. However, as discussed earlier, this is not accurate. Temperature primarily measures the emission level related to the potential energy of a system.

In the case of a phase transition, such as from ice to water, molecules are liberated from their fixed positions within hydrogen bonds, and their kinetic energy must increase through the process. Consequently, if temperature accurately reflected the average kinetic energy of a system, it would be expected to rise during the transition. However, the temperature remains constant during the transition. This contradiction arises because temperature is not directly associated with kinetic energy but with potential energy.

During a phase transition, the temperature remains constant because it is constrained by the potential energy limits of hydrogen bonds until the bonds break. As latent heat is added to the system, the bonds expand to accommodate more potential energy. Simultaneously, the strength of the bonds decreases as they are stretched, and the average intensity of transimpacts increases, making the bonds more vulnerable. Eventually, the phase transition occurs as the bonds are broken by transimpacts. The potential energy of these bonds is limited by the intensity of transimpacts that the bonds can withstand.

As demonstrated earlier, the energy exchange between different components occurs through transimpacts, maintaining equilibrium, particularly in their emission levels. If one component has a higher level of potential energy, the exchange will eventually balance it to a common level, and vice versa. Essentially, the emission level of the entire system is anchored by the potential energy limits of the bonds. Therefore, because the system's potential energy level is fixed, the temperature, measured by the emission level of the bonds, remains constant during a phase transition.

Conclusions

Transimpact is the underlying mechanism behind many physical interactions and phenomena, including the vibration of molecules in solids, the random motion of molecules in fluids, the Brownian motion of small particles, the rotation of Crookes radiometers, and phase transitions, among others. The random movement of molecules does not have sufficient momentum to drive Brownian motion; instead, Brownian motion is driven by transimpact. Temperature is typically a measure of the average potential energy level, rather than the kinetic energy of a system. Different forms of energy can be exchanged through transimpacts to maintain a certain balance within a system. These equilibrium processes create the delusion of a direct correlation between temperature and kinetic energy.

Revision History

- 06/16/2019: Initial Post on Stanford Site
- 11/01/2025: Published on Zenodo
- 12/17/2025: Adding Links to Summaries of Related Articles

Links to Summaries of Related Articles

- https://cs.stanford.edu/people/zjl/abstract.html, PDF
- https://sites.google.com/view/zjl/abstracts, PDF
- https://xenon.stanford.edu/~zjl/abstract.html, PDF
- https://doi.org/10.5281/zenodo.17967154, PDF

Further Literature

- Misconceptions in Thermodynamics (PDF: DOI) (中文: DOI)
- The Mechanism Driving Crookes Radiometers (PDF: DOI) (中文: DOI)
- The Cause of Brownian Motion (PDF: DOI) (中文: DOI)
- Can Temperature Represent Average Kinetic Energy? (PDF: DOI) (中文: DOI)
- The Nature of Absolute Zero Temperature (PDF: DOI) (中文: DOI)

- The Triangle of Energy Transformation (PDF: DOI) (中文: DOI)
- Is Thermal Expansion Due to Particle Vibration? (PDF: DOI) (中文: DOI)
- Superfluids Are Not Fluids (PDF: DOI) (中文: DOI)
- Why a Phase Transition Temperature Remains Constant (PDF: DOI) (中文: DOI)
- What Causes Friction to Produce Heat? (PDF: DOI) (中文: DOI)
- The Easiest Way to Grasp Entropy (PDF: DOI) (中文: DOI)
- Entropy Can Decrease (PDF: DOI) (中文: DOI)
- The Restoration Principle (PDF: DOI) (中文: DOI)
- Is There a Sea of Free Electrons in Metals? (PDF: DOI) (中文: DOI)
- Electron Tunnel (PDF: DOI) (中文: DOI)
- Unified Theory of Low and High-Temperature Superconductivity (PDF: DOI) (中文: DOI)
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- The Simplest Derivation of $E = mc^2$ (PDF: DOI) (中文: DOI)
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