How to Understand Relativity

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Constant Speed of Light

Observations provide insight into the natural laws that govern the world. To explain these observations, physical models or theories are developed. The theory of special relativity, for example, was formulated by Albert Einstein to account for the observation that the speed of light is constant. Thus, the core concept of special relativity lies in understanding the invariance of the speed of light.

In classical physics at low speeds, velocity is relative to the observer and additive. For example, if a person throws a football at 10 meters per second on a train moving at 20 meters per second, an observer on the ground would measure the ball's speed as 30 meters per second. For much of history, it was believed that light traveled through a hypothetical medium called the luminiferous aether, which was thought to permeate space and serve as the carrier of light waves. According to this view, the speed of light, like other velocities, was also additive. Based on this assumption, one might expect the speed of light to vary depending on its direction relative to the Earth's motion, with a different speed in the direction of the Earth's rotation compared to a perpendicular direction.

In 1887, Michelson and Morley conducted a highly sophisticated experiment to detect the presence of the luminiferous aether. To the surprise of many, they found no significant difference in the speed of light when measured in the direction of Earth's motion through the presumed aether, compared to when measured at right angles to it. This result provided the first strong evidence against the widely accepted theory of a stationary aether, suggesting instead that the speed of light is constant, independent of both the light source and the observer.

Based on observations, Isaac Newton claimed that different colors of light traveled at the same speed. The speed of electromagnetic waves is predicted in Maxwell's Equations and determined only by the constants of electricity and magnetism:

$$(1) \qquad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299,792,458\ m/s$$

where ε_0 is the electric permittivity and μ_0 is the magnetic permeability. This is the same speed as light, suggesting that light is an electromagnetic wave. The constant speed of electromagnetic waves is determined independently of any specific reference frame or observer, which aligns with the results of Michelson and Morley's experiment. These observations collectively indicate that the speed of light is constant, regardless of the motion of the light source or the observer. Recognizing the invariance of the speed of light is the fundamental starting point for understanding special relativity.

Time Dilation

Now that, if the speed of light is rigid, space or time must yield. In physics, speed is defined and determined by two fundamental quantities: time and space, where space is typically measured in distance.

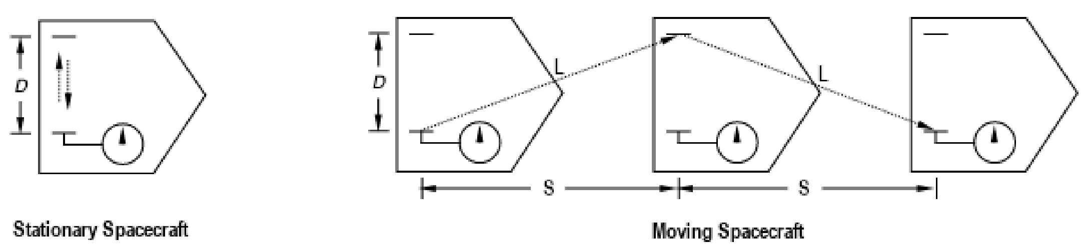
(2) $$speed = \frac{distance}{time}$$

In Newtonian physics, time and space are considered absolute and rigid, making it counterintuitive to think that they could vary for different observers. However, to reconcile the speed definition in formula (2) with the observed constancy of the speed of light, the time and space on the right side of the equation must yield.

Then, what is the definition of time? Time is conventionally measured by the interval between events. For instance, a day is defined as one complete rotation of the Earth, which is further divided into 24 hours. More precise measurements of time are based on the vibration period of certain crystals or the transition period of atomic electrons. All of these time measurements rely on the implicit assumption of a consistent speed for certain processes. Essentially, when a physical phenomenon occurs at a given speed, the time interval or period between two events can be determined using the distance, as given by:

(3) $$time = \frac{distance}{speed}$$

This is just a mutation of Equation (2); however, it defines the time in terms of speed. With the universal constant speed of light, time can be consistently defined and determined using Equation (3). Hence, the time dilation of a moving object can be easily demonstrated. Consider a simple photon clock consisting of two mirrors vertically separated by a distance D, between which a photon bounces back and forth. The clock ticks once each time the photon hits the bottom mirror. One such clock is installed in a stationary spacecraft on the ground, as shown in the left part of the following figure. Another is fixed in a spacecraft moving at a speed v relative to the stationary spacecraft, as shown in the right part of the figure.



For a "stationary" observer on the ground, the period of a clock tick is the time the photon travels a distance of 2D in the stationary spacecraft:

(4) $$\Delta t_0 = \frac{2D}{c}$$

Here, c is the speed of light and Δt_0 is the ticking period of the stationary clock. An observer inside the moving spacecraft will also calculate the period of the clock inside the craft using the same equation as (4).

To a stationary observer on the ground, the photon path is longer on the moving clock in the spacecraft. A ticking period of the moving clock is the total time for the photon to travel through the diagonal path 2L:

$$(5) \qquad \Delta t = \frac{2L}{c}$$

For the same tick, to the observer inside the spacecraft, the photon travels 2D and the period is Δt_0, computed with equation (4), which is shorter than Δt computed by the stationary observer. That is to say, the moving clock will be running slower than the stationary clock on the ground.

With the Pythagorean theorem,

$$(6) \qquad L^2 = D^2 + S^2$$

Here, S is the distance the spacecraft traveled during period ½Δt:

$$(7) \qquad S = \frac{1}{2}\Delta t v$$

Now, Δt can be found by eliminating D, L, and S from these four equations:

$$(8) \qquad \Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

This shows that the moving clock is slower than the stationary clock. This is the time dilation in a moving reference frame.

Space Contraction

Space contracts in a moving reference frame. Assume a stationary observer on the ground measures the spacecraft traveling L_0 meters during a tick on the clock in the spacecraft. A tick on the clock in the spacecraft takes Δt_0 time for an observer in the spacecraft, given in equation (4), and Δt for the observer on the ground, given in equation (5). What will be the travel distance L during the same period for the observer in the spacecraft? Since both observers are in inertial reference frames, both observers should agree and observe the same travel speed of the spacecraft during the same process:

$$(9) \qquad \frac{L}{\Delta t_0} = \frac{L_0}{\Delta t}$$

Here, the right side of the equation is the speed calculated by the stationary observer on the ground, and the left side is computed by the moving observer in the spacecraft. With the time dilation in equation (8), we can find the space contraction:

(10) $$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

Relativistic Mass

Mass is also relative. Let's consider a ball that is dropped inside a spacecraft. The ball travels a vertical distance of S meters during a tick on the local clock inside the spacecraft. To an observer inside the spacecraft, the momentum of the ball is given by equation (11):

(11) $$p_0 = m_0 v_0 = m_0 \frac{S}{\Delta t_0}$$

Here, m_0 is the rest mass of the ball, v_0 is the velocity of the ball, and Δt_0 is the period of the local clock inside the spacecraft.

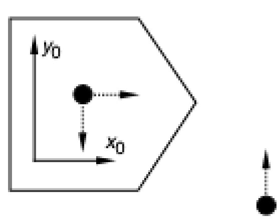
To a stationary observer on the ground, the vertical momentum of the ball is given by equation (12):

(12) $$p = mv = m\frac{S}{\Delta t}$$

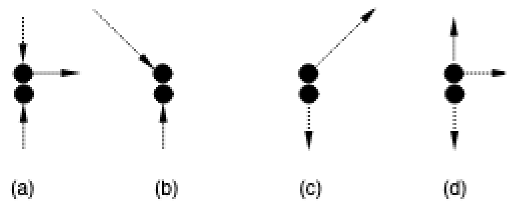
Here, m is the relativistic mass of the ball, v is the vertical velocity of the ball, and Δt is the period of the moving clock. The vertical distance S is the same for both observers, but the time duration is dilated to Δt for the observer on the ground. Since both observers are in inertial reference frames, disregarding the horizontal component, both should observe the same vertical momentum for the ball during the dropping process as in equation (13):

(13) $$m_0 \frac{S}{\Delta t_0} = m\frac{S}{\Delta t}$$

Equation (13) can also be derived from a different perspective. Let's shoot a ball with the same rest mass and velocity as the one that was dropped earlier, but in the opposite direction, so that the two balls will collide in the middle, as shown in the figure below.



The momentum of the upward ball will be the same as that given in equation (11) to the ground observer, but in the opposite direction. After the collision, disregarding the horizontal component, both balls will bounce back vertically with the same vertical momentum as they started, but in the opposite direction, as shown in the following figure.



Using the time dilation formula (8), we can simplify equation (13) as

$$(14) \qquad m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

This shows that the moving object becomes heavier.

Summary

All measurements of physical properties that are stationary relative to an observer are the same as in Newtonian physics and are referred to as proper measurements. Relativistic measurements, on the other hand, involve measuring the properties of objects in motion. For example, if you observe a person sipping a cup of coffee in a high-speed spacecraft, you would perceive his actions as slowed down. However, if you were in the same spacecraft, you would perceive his behavior as normal, since both you and the person are moving at the same speed.

Revision History

- 09/17/2019: Initial Post on Stanford Site
- 11/02/2025: Published on Zenodo
- 12/17/2025: Adding Links to Summaries of Related Articles

Links to Summaries of Related Articles

- https://cs.stanford.edu/people/zjl/abstract.html, PDF
- https://sites.google.com/view/zjl/abstracts, PDF
- https://xenon.stanford.edu/~zjl/abstract.html, PDF
- https://doi.org/10.5281/zenodo.17967154, PDF

Further Literature

- Misconceptions in Thermodynamics (PDF: DOI) (中文: DOI)
- The Mechanism Driving Crookes Radiometers (PDF: DOI) (中文: DOI)

- The Cause of Brownian Motion (PDF: DOI) (中文: DOI)
- Can Temperature Represent Average Kinetic Energy? (PDF: DOI) (中文: DOI)
- The Nature of Absolute Zero Temperature (PDF: DOI) (中文: DOI)
- The Triangle of Energy Transformation (PDF: DOI) (中文: DOI)
- Is Thermal Expansion Due to Particle Vibration? (PDF: DOI) (中文: DOI)
- Superfluids Are Not Fluids (PDF: DOI) (中文: DOI)
- Why a Phase Transition Temperature Remains Constant (PDF: DOI) (中文: DOI)
- What Causes Friction to Produce Heat? (PDF: DOI) (中文: DOI)
- The Easiest Way to Grasp Entropy (PDF: DOI) (中文: DOI)
- Entropy Can Decrease (PDF: DOI) (中文: DOI)
- The Restoration Principle (PDF: DOI) (中文: DOI)
- Is There a Sea of Free Electrons in Metals? (PDF: DOI) (中文: DOI)
- Electron Tunnel (PDF: DOI) (中文: DOI)
- Unified Theory of Low and High-Temperature Superconductivity (PDF: DOI) (中文: DOI)
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- The Evolution from the Law of Gravitation to General Relativity (PDF: DOI) (中文: DOI)
- The Simplest Derivation of $E = mc^2$ (PDF: DOI) (中文: DOI)
- How to Understand Relativity (PDF: DOI) (中文: DOI)
- Mathematics Is Not Science (PDF: DOI) (中文: DOI)
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