

# Mathematics Is Not Science

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## Abstract

Mathematics is a language. By analogy with natural language, one may regard *axioms* as vocabulary, *operational rules* as grammar, and *theorems* as condensed idioms. Science aims to reveal truths about the world, along with the methodologies for acquiring such knowledge. Although scientific knowledge can be expressed in natural languages, mathematics offers a more precise and concise mode of expression—and often a more powerful instrument for extending that knowledge. In essence, mathematics is not a science. Mathematical theorems are infallible, whereas scientific theories must be falsifiable. Nevertheless, mathematics serves both as a precise language for articulating scientific understanding and as a powerful instrument for uncovering equivalent sets of knowledge through abstraction and extension.

## Questions

Mathematics plays a compelling role in scientific research, which raises a profound question: **Can mathematics ever replace science?** More specifically, is it possible to deduce the laws and properties of physics purely through mathematical reasoning, without any need for physical observation? For example, Maxwell's equations predict that electromagnetic waves propagate at a characteristic speed—one that is identified as the speed of light. Yet could this value ever be derived *purely* from mathematics, without any empirical measurement? If the answer is no, then what exactly is the role that mathematics plays within scientific practice?

## Overview

Mathematics is crucial to science, but optional. While mathematics provides succinct and precise language and a powerful tool in scientific research, it gives the misleading impression that science is fundamentally based on mathematics. In reality, the foundation of science lies in observations, not mathematics. Scientific knowledge can be expressed in natural language, although this tends to be less concise. Indeed, many sciences still cannot be described mathematically.

Mathematics is not a science. It enriches scientific understanding by providing rigorous transformations based on existing observations and knowledge. However, scientific predictions or mathematical extensions must be validated through further empirical observation. All fundamental scientific theories, principles, and laws originate from observation and cannot be deduced solely through mathematics. Therefore, while mathematics is essential to science, it is not a scientific discipline in itself.

More importantly, mathematics is infallible, but science is inherently empirical and falsifiable. Any scientific theory, whether developed from observations or derived from existing knowledge, is ultimately grounded in, and must be verified by, empirical observations, making it inherently subject to falsification.

Science cannot be derived purely through mathematical deduction. While many scientific discoveries arise from mathematical extensions, these extensions are always grounded in empirical evidence. For example, the speed of light can be derived from Maxwell's equations, but it is a misconception to think this result comes purely from mathematics. In reality, it is a mathematical extension of existing scientific knowledge, based on empirical measurements of vacuum permittivity and vacuum permeability—quantities determined through observation, not mathematical deduction.

Mathematical theorems are derived from precisely defined axioms and operational rules, and mathematics remains self-consistent within this formal framework. When applied to scientific reasoning, observational data must first be translated into appropriate assumptions or axioms before mathematical theorems can be meaningfully used to extend existing knowledge. Indeed, mathematics itself was originally abstracted from observations and developed as a tool to support scientific practice and inquiry.

These insights help us identify equivalent knowledge, which extends our understanding through mathematical inferences and deductions. They also reveal the layered structure of knowledge, clarify the reliability of each layer within the system, and propose the concept of a foundational set of principles from which other knowledge can ultimately be derived. This article aims to elucidate these concepts.

### **Mathematics is Infallible**

The goal of mathematics is to provide a systematic and logical framework for understanding and describing patterns, relationships, and structures within abstract systems. Mathematics relies on precisely defined axioms and operational rules. Mathematical theorems are rigorously proven through pure reasoning, adhering to the rules derived from axioms and previously established results. Therefore, mathematics maintains self-consistency within its defined domain of axioms and operations, ensuring its reliability.

For example, Euclidean geometry is a self-consistent mathematical system built on a small set of intuitive postulates, known as the five axioms, defined on a flat plane. All subsequent propositions (theorems) derived from these axioms are logically valid within that plane. These theorems can be applied to scientific contexts, provided the conditions align with a flat, two-dimensional space. For instance, when two orthogonal forces act on an object, their combined effect can be calculated using the Pythagorean theorem.

Despite its limitation of assuming a flat plane, Euclidean geometry maintains internal coherence within its defined domain and provides a reliable approximation for short distances on curved surfaces. Various other self-consistent non-Euclidean geometries have been developed; Riemannian geometry, in particular, serves as the mathematical framework of Einstein's theory of general relativity in the context of curved space. The key principle is that mathematical systems remain internally consistent within their assumptions. As long as these assumptions are applied, their theorems remain valid in their applications.

Mathematics can be used to extend scientific knowledge because its proofs are rigorous and grounded in pure reasoning, such as inductive and deductive methods. For example, to prove that a mathematical statement (or theorem) involving a natural number  $n$  holds for all values of  $n$ , a proof by induction involves two main steps:

1. The base case: Prove that the statement holds for 0.
2. The induction step: Prove that for an arbitrary number  $n$ , if the statement holds for  $n$ , it holds for  $n + 1$ .

With the proof of these two steps, the truth of the statement can be established using deductive reasoning. First, the validity of step 2 serves as the general premise: if the statement holds for an arbitrary natural number  $n$ , then it holds for  $n + 1$ . Since the statement holds for 0, as proved for a special case in step 1, the statement will hold for 1 ( $0 + 1$ ), provided in the general premise as proved in step 2. This reasoning can be applied recursively to prove the statement for 2, 3, and so on, thereby proving it for all natural numbers.

For instance, to prove that statement

$$(1) 1 + 3 + 5 + \dots + (2n + 1) = (n + 1)^2$$

is valid for all natural numbers, using inductive reasoning involves:

1. Prove the statement (1) holds for  $n = 0$ , which is true in this case because (1) reduces to  $(2 \times 0 + 1) = (0 + 1)^2$ , and
2. If the statement holds for an arbitrary number  $n$ , the statement also holds for  $n + 1$ .

To prove step 2, let's assume that:

$$(2) 1 + 3 + 5 + \dots + (2n + 1) = (n + 1)^2$$

holds for an arbitrary number  $n$ . Then, we need to prove it holds for  $n+1$ , which is

$$(3) 1 + 3 + 5 + \dots + (2n + 1) + (2(n + 1) + 1) = ((n + 1) + 1)^2$$

With assumption (2), the left side of (3) is

$$(4) 1 + 3 + 5 + \dots + (2n + 1) + (2(n + 1) + 1) = (n + 1)^2 + (2(n + 1) + 1) = ((n + 1) + 1)^2$$

This completes the second part of step 2, showing that the statement also holds for  $n+1$ . With both steps established, the inductive reasoning is complete. From this, deductive logic confirms that statement (1) holds for all natural numbers. This example illustrates the rigor and self-consistency of mathematical reasoning—once a theorem is proven, its validity is absolute.

Mathematics's rigorous and self-consistent nature makes it a succinct language and a powerful tool for science. However, it is not always necessary for scientific practices. For instance, Rutherford's discovery of the atomic nucleus was a fundamental scientific breakthrough that was not expressed mathematically. Similarly, Newton's law of universal gravitation

can be fully articulated using natural language. Many scientific concepts were originally described in this manner, and this practice continues in various scientific disciplines today.

Nevertheless, mathematics allows complex scientific ideas to be expressed concisely and precisely. When feasible, scientific concepts and theories should be articulated more succinctly through mathematics, as exemplified by the law of gravitation in the following formula:

$$(5) F = G \frac{m_1 m_2}{r^2}$$

Here,  $F$  is the gravitational force acting between two objects,  $m_1$  and  $m_2$  are the masses of the objects,  $r$  is the distance between the centers of their masses, and  $G$  is the gravitational constant. This example demonstrates the succinctness of the law of gravitation when expressed mathematically. More importantly, this formulation allows for the straightforward study and extension of the behavior and orbits of celestial bodies. Consequently, many branches of mathematics have been developed to address these needs.

### **Scientific Theories must be Falsifiable**

People often equate science with truth. However, science is not truth itself, but a systematic method for seeking it. Scientific theories must be falsifiable, that is, capable of being tested and potentially disproven through observation. Science aims to understand the natural world through observation, experimentation, and the construction of explanations for how it operates. By uncovering the underlying principles and laws that govern the universe, science develops models and theories that explain and predict natural phenomena. For these models to be considered scientific, they must yield testable, falsifiable predictions. Without this criterion, a theory falls outside the realm of science.

Like mathematics, both inductive and deductive reasoning are fundamental to scientific research. Scientific models are built using these two methodologies and can generally be classified as either inductive or deductive. Inductive reasoning derives general principles from empirical observations, while deductive reasoning extends those principles to generate new knowledge, often through mathematical derivation. Some scientists favor one approach over the other, leading to distinctions such as experimental physicists who prioritize observation to uncover physical laws and theoretical physicists who rely on deduction to expand existing knowledge frameworks.

Although mathematics is infallible and essential to science, it is not a scientific discipline because science is inherently falsifiable. Unlike mathematics, the application of these methodologies in science cannot be numerically formalized with the same level of rigor. For instance, after observing that one, two, or even billions of swans are white, one might use inductive reasoning to hypothesize that all swans are white. Consequently, using deductive reasoning, one might propose a theory that all swan feathers are white. However, a single black swan would disprove both the initial hypothesis and the deductive theory, demonstrating the limitations of these methodologies in scientific practice.

No matter how solid today's scientific knowledge may appear, it is ultimately built, whether through inductive or deductive reasoning, on the foundation of incomplete observations, and therefore cannot be considered entirely certain. This inherent uncertainty makes the scientific knowledge system fundamentally falsifiable. For example, Isaac Newton formulated the

law of universal gravitation based on the observations and data available in his time, expressing it in his famous formula (5). This law not only accounted for Kepler's three empirical laws of planetary motion but was also confirmed by extensive observations, establishing it as a cornerstone in our understanding of celestial mechanics. The development of this law illustrates the essential role of inductive reasoning in scientific practice.

However, Newton's law of gravitation faced a significant challenge in explaining the precession of Mercury's perihelion. This longstanding problem puzzled astronomers for over 250 years in studying the Solar System. It was eventually addressed by Einstein's theory of general relativity. General relativity is a classic example of deductive reasoning, building on existing knowledge and the equivalence assumption of inertial and gravitational masses. As other predictions of Einstein's theory were further confirmed, it gained widespread acceptance and provided a more accurate understanding of the universe. Nonetheless, future observations may reveal that this theory is not as precise as currently believed.

Similarly, laws and theories that are deductively derived from established scientific results ultimately depend on the validity of the fundamental knowledge from which they originate. Fundamental laws and principles are inherently empirical and cannot be proven with the same certainty as mathematical theorems. Therefore, all scientific knowledge is built upon inductive reasoning and shares the same falsifiable nature. Regardless of how robust a scientific theory may appear, it remains subject to scrutiny through future observations and is inherently falsifiable.

This inherent falsifiability is not limited to the law of gravitation or general relativity; it applies to all scientific laws, theories, and principles, including Newton's second law of motion, Planck's law of black-body radiation, Coulomb's law of electrostatic force, and Maxwell's equations of electromagnetic waves. Each of these was formulated based on the observations available at the time and reflects only a partial view of the underlying physical reality. Numerous unexplained phenomena in quantum physics underscore the limitations of our current understanding of the physical world. Although scientific knowledge continues to advance toward a more accurate representation of reality, there is no guarantee that we will ever attain a complete or final picture. Furthermore, the precision of observational data and the assumptions embedded within scientific models are themselves subject to inherent limitations.

On the other hand, can fundamental scientific knowledge be derived purely through mathematical reasoning without any physical observation? Take the speed of light, for example; it can be derived from Maxwell's equations, but it is a misconception to regard this as a purely mathematical result. In reality, the derivation relies on empirical constants: vacuum permittivity and vacuum permeability, both of which are determined through physical measurements, not mathematical deduction. While many scientific breakthroughs emerge from mathematical extrapolation, such extrapolations are always grounded in empirical evidence. Similarly, mathematical constants such as  $e$  and the imaginary unit  $i$  are purely abstract, but they gain physical significance only through their applications in scientific contexts. Therefore, no scientific law or theory can be derived solely through mathematics without being anchored in foundational empirical knowledge.

The ultimate goal of science is to uncover the underlying physics behind observations of the world. Scientific laws and theories are developed from these observations. They must explain these observations and, more importantly, make predictions about future observations. The verifiability of these predictions ensures that the laws and theories can be tested and potentially falsified. For instance, verifying the predictions of the law of universal gravitation, which states that "all objects attract each other", would require observing the behaviors of all objects, an impossible task. However, falsifiability

requires only one anomalous instance to disprove the theory. For example, do photons gravitationally attract each other? Falsifiability is a fundamental characteristic of all scientific disciplines.

### Equivalent Knowledge

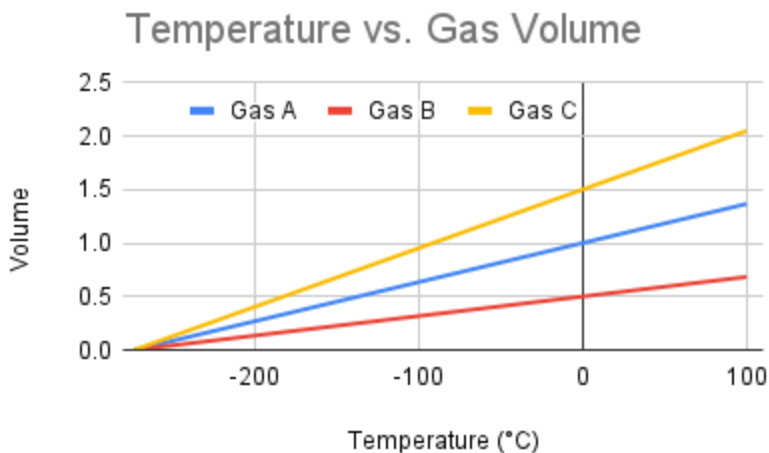
The use of mathematics in scientific practices greatly facilitates and accelerates scientific progress. One of its most notable advantages is its ability to expand existing knowledge through deductive reasoning. The concept of "equivalent knowledge" refers to the ability to derive additional insights about a system, such as its properties, principles, theories, and laws, using existing scientific understanding, often through mathematical deduction. Mathematics proves particularly useful and powerful in this context, as it allows for the systematic and precise extension of knowledge based on established principles.

For instance, a line is uniquely determined by two points mathematically. In this context, the line and the two points convey the same information. For example, points (0, 1) and (100, 1.3661) define the line described by the equation:

$$(6) y = \frac{1}{273.15}x + 1$$

Equation (6) is equivalent to the information provided by these two points. Any two points on this line can be used to determine the line represented by equation (6). Therefore, any pair of points on the line is essentially equivalent knowledge in terms of defining the line. In particular, it also infers that point (-273.15, 0) is on the line and can also determine the same line with any one of the initial points. Mathematically, this point is the equivalent knowledge derived from the initial two points.

Now, let's apply this middle school mathematics to physics. If x denotes the temperature of a gas and y represents the volume of the gas observed at that temperature, then line (6) describes the relationship between the temperature and volume of the gas, indicating that the volume increases with temperature. The two initial points in question represent the volumes of the gas at temperatures of 0°C and 100°C, respectively.



Based on this mathematical relationship, the point  $(-273.15, 0)$  suggests the volume would be zero when the temperature is  $-273.15^{\circ}\text{C}$ . Since a gas volume cannot decrease below this point, this temperature must represent the minimum temperature of the gas, which is referred to as absolute zero. Thus, absolute zero is an equivalent piece of knowledge derived from the observations of gas behavior. Subsequent observations indicate that the temperature-volume relationship for various gases also predicts this same minimum temperature as the volume approaches zero, as illustrated in the figure above, confirming the validity of this mathematical inference.

Another example of mathematical equivalence in physics applications is the relationship between a function's time domain and frequency domain representations via the Fourier transform. The characteristics of a function can be transformed from the time domain to the frequency domain, and conversely, the original function can be reconstructed from its frequency components. Essentially, these two sets of information are mathematically equivalent; they are simply different representations of the same underlying data. Leveraging this equivalence, AI can efficiently simulate a person's voice by extracting its frequency characteristics from just a short recording of his speech.

Similarly, Newton's law of universal gravitation can be used to predict Kepler's three empirical laws of planetary motion. Conversely, Kepler's laws can be used to derive the law of universal gravitation, demonstrating their equivalence through mathematical transformations. Many fields of scientific knowledge are extended in this way through mathematics. Indeed, making predictions based on existing theories has been a primary scientific endeavor, with the majority of knowledge being extended in this manner.

When a scientific theory is expressed in mathematical form, its mathematical extrapolations effectively produce predictions that are firmly rooted in established knowledge. The inherent rigor and internal consistency of mathematics ensure that if a theory is valid for a system under specific conditions, then the conclusions derived from it through mathematical reasoning remain valid under those same conditions. This is because the extended knowledge is mathematically equivalent to the original theory, offering a different representation of the same underlying physical reality. In essence, mathematics functions as a powerful tool that extends a limited set of scientific insights into a broader and more integrated understanding of a system.

## **The Structure of Knowledge**

A domain of knowledge about a system encompasses all understandings of its physics, with scientific knowledge representing a subset within this domain. Within this subset, not all knowledge is equally reliable. Knowledge can be organized into layers, with the base layer consisting of direct observations, which are the most trustworthy as they directly reflect the underlying physics. These observations are typically reproducible under the same conditions.

The next layer consists of knowledge derived from observations. The scientific laws, theories, and principles in this layer may be mathematically formulated but generally lack formal mathematical proof due to the absence of a foundational mathematical base. Although these concepts cannot be rigorously proven mathematically, their predictions can be verified through observations. Examples include Newton's law of universal gravitation, Newton's second law of motion, Maxwell's equations, Ohm's law, and Charles's law. These can be considered scientific axioms and form the foundation for subsequent layers of knowledge.

Subsequent layers of scientific knowledge, built on these foundational laws and theories through mathematical inference or deduction, are also subject to verification through observations. These layers are generally considered less reliable than the foundational ones, as they often involve more complex assumptions and approximations, such as in the kinetic theory of gases and the BCS theory of superconductivity. As we progress through these layers, the trustworthiness of the knowledge typically decreases, as each layer relies on the accuracy of the underlying principles, which may introduce potential flaws or inaccuracies.

The equivalent knowledge discussed previously usually pertains to the third layer or above, which is mathematically derived from the foundational layer. Although the principle of equivalent knowledge demonstrates a significant application of mathematics to expand scientific understanding, the equivalence is purely mathematical. The validity of this extended knowledge relies entirely on the accuracy of the original foundational knowledge. Therefore, if the foundational knowledge is flawed, the extended knowledge will also be flawed.

For example, the BCS theory is commonly used to explain superconductivity at low temperatures. However, it faces challenges with recent discoveries of superconductors that function at higher temperatures. The BCS theory is based on the collision model of electrical resistance, which relies on concepts such as metallic bonds and the sea of free-moving electrons. The validity of the BCS theory depends on the accuracy of these underlying concepts. If any of these concepts are flawed, the BCS theory itself would be undermined.

Both the BCS theory and the collision model of resistance fail to account for behavior at high pressures, suggesting the issue may lie with the fundamental concepts underlying these theories. A recent study posed the question: "[Is There a Sea of Free Electrons in Metals?](#)" According to the metallic bond model, metals are held together by a sea of free electrons. If this model were correct, removing free electrons from a metal would cause the cations, which constitute the lattice, to repel each other, leading to the disintegration of the conductor's structure. This raises a critical question: How can the structure of conductors remain stable with such a seemingly unstable "glue", especially when electrons are actively flowing through the conductors as electric currents?

While foundational knowledge is generally robust, it is not without its flaws. Indeed, imperfections in foundational theories are not uncommon. For example, the geocentric model, which placed the Earth at the center of the universe, was once the prevailing view for thousands of years across many ancient civilizations. This model was so entrenched that when Copernicus proposed his heliocentric theory in the early 1500s, it was initially dismissed as a radical idea.

Although science has advanced significantly over the past 500 years, this does not mean that our foundational knowledge is without flaws. Continuous corrections and refinements are essential to scientific progress. For example, it was once believed that the continents were fixed in place relative to each other. Alfred Lothar Wegener's 1912 hypothesis of continental drift was initially rejected by mainstream geologists. It was not until the 1950s, when discoveries such as paleomagnetism provided strong supporting evidence, that the theory of continental drift gained widespread acceptance and became a cornerstone of modern plate tectonics.

Refinements and corrections are quite common even in the most fundamental areas of knowledge. For example, Newton's law of universal gravitation was formulated based on the observations and understanding available at the time, including Kepler's laws of planetary motion, which did not account for and could not explain the precession of Mercury's orbit.

Consequently, the law's predictive accuracy was limited by the constraints of existing knowledge. These limitations were later addressed by the more comprehensive theory of general relativity.

These examples illustrate that scientific knowledge is never perfect. Scientists are tasked with challenging existing theories and building knowledge on a more solid foundation. They also highlight two crucial aspects of equivalent knowledge. First, mathematics alone cannot replace science; the validity of extended knowledge relies on the quality of the original theories. Second, equivalent extensions offer further opportunities to verify existing knowledge, which can either reinforce or challenge the original understanding. For instance, the precession of Mercury's orbit revealed limitations in the law of gravitation, which became a new observation in scientific inquiry. Ultimately, this led to the development of a more accurate understanding through general relativity.

### Mathematical Extension of Knowledge

Nevertheless, many elegant models and theories have significantly advanced our understanding. For example, the positron (or antielectron) was predicted by the Dirac equation and later confirmed through numerous experiments and observations. While it might appear that the positron was predicted solely through mathematical equations, this is a misconception. The Dirac equation was developed based on existing knowledge, which itself was derived from other fundamental observations and understandings. Essentially, the Dirac equation provides a new perspective on this original knowledge. Without the foundational knowledge, the Dirac equation could not have been formulated.

Similarly, the constant speed of light, an intrinsic property of electromagnetic waves, can be predicted by Maxwell's equations from other properties, particularly the vacuum permittivity  $\epsilon_0$  and the vacuum permeability  $\mu_0$ :

$$(7) c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

This has been reinforced and confirmed by numerous experiments, most notably the Michelson-Morley experiment. The theory of special relativity is essentially a mathematical extension based on the constant property of the speed of light. Maintaining the constancy of light speed requires adjustments in both space and time since speed is defined as distance divided by time:

$$(8) \text{ speed} = \frac{\text{space}}{\text{time}}$$

Thus, the properties of space contraction and time dilation are extended through mathematical transformations from the fundamental property of constant light speed. Mathematically, Equation (8) is equivalent to:

$$(9) \text{ space} = \text{speed} \times \text{time}$$

While Einstein's relativity theory usually explains space contraction and time dilation through Lorentz transformations, Equation (9) offers a simpler understanding of these phenomena. To uphold the constant speed of light according to this equation, as time dilates, space must contract. This raises the question: How far can this reasoning be extended? As

speed approaches that of light, time dilates towards zero, suggesting that space, as per Equation (9), must also contract to zero. This potential singularity is purely a mathematical prediction—a phenomenon not yet observed.

Taking this thought a step further: what would it imply if time were to become negative, a mathematically valid concept? Could space similarly assume negative values, and what would negative space signify? While these predictions are grounded in mathematics, their confirmation depends on empirical observations in science. These represent equivalent knowledge introduced by mathematics, but pending scientific verification.

It is crucial to recognize that if the base knowledge is flawed, any extended knowledge built upon it will also be incorrect. Therefore, a solid base of accurate knowledge is essential for all scientific endeavors. Scientists play a critical role not only in expanding knowledge but, more importantly, in rigorously challenging and verifying existing knowledge to ensure its accuracy and extend knowledge.

### **The Minimum Set of Knowledge**

Consider all properties of electromagnetic waves as elements within a set. Maxwell's equations can be understood as operations on this set that project one subset of knowledge onto another. For example, formula (7) demonstrates how the speed of light is derived from these equations, representing an equivalent of knowledge based on other understandings. Similarly, space and time are properties within a domain where the speed of light also resides as a constant property. Einstein's theory of special relativity and the Lorentz transformations act as operations within this domain. Therefore, time dilation and space contraction are extensions and equivalent knowledge within this domain, projected from the constant property of light speed.

Consider a broader scope of the knowledge domain encompassing electromagnetic wave properties alongside space and time, with Maxwell's equations and Lorentz transformations as operations. The constant speed of light can be derived from the properties of electromagnetic waves described by Maxwell's equations. Space contraction and time dilation, in turn, stem from the constancy of light speed using Lorentz transformations. Therefore, special relativity is equivalent knowledge derived from a small subset of electromagnetic wave properties. In other words, special relativity is a mathematical realization that extends from the understanding of electromagnetic waves. Thus, the entire breadth of this knowledge domain expands from a fundamental subset, representing the minimal set of knowledge in this domain.

Generally, natural laws and theories function as operations within a given domain of knowledge, facilitating predictions and extensions from one set of knowledge to another. While it seems that mathematics enables us to expand our knowledge, this extension is simply the realization of equivalent subsets within the same domain. Within any domain of knowledge, there exists a minimum subset from which all other knowledge can be derived using available operations, such as scientific theories and laws. In the previous example, the minimum set of knowledge comprises the Lorentz transformations, Maxwell's equations, and the properties of electromagnetic waves. The comprehension of the constant speed of light, space contraction, and time dilation represents equivalent knowledge extended from this minimum set.

It appears that our understanding of the universe is divided into distinct domains. As knowledge advances, the boundaries between these domains become less defined and begin to overlap. For example, chemistry and physics were once

separate disciplines but are now seen to share significant common ground, forming interconnected domains within a larger realm of knowledge.

We envision that understanding the entire universe is an interconnected domain of knowledge that can be derived from a minimum set of fundamental knowledge. This fundamental set reflects the essential properties and operations of the universe.

### Revision History

- 10/15/2024: Initial Post on Stanford Site
- [11/02/2025: Published on Zenodo](#)
- [12/17/2025: Adding Links to Summaries of Related Articles](#)

### Links to Summaries of Related Articles

- <https://cs.stanford.edu/people/zjl/abstract.html>, PDF
- <https://sites.google.com/view/zjl/abstracts>, PDF
- <https://xenon.stanford.edu/~zjl/abstract.html>, PDF
- <https://doi.org/10.5281/zenodo.17967154>, PDF

### Further Literature

- [Misconceptions in Thermodynamics \(PDF: DOI\) \(中文: DOI\)](#)
- [The Mechanism Driving Crookes Radiometers \(PDF: DOI\) \(中文: DOI\)](#)
- [The Cause of Brownian Motion \(PDF: DOI\) \(中文: DOI\)](#)
- [Can Temperature Represent Average Kinetic Energy? \(PDF: DOI\) \(中文: DOI\)](#)
- [The Nature of Absolute Zero Temperature \(PDF: DOI\) \(中文: DOI\)](#)
- [The Triangle of Energy Transformation \(PDF: DOI\) \(中文: DOI\)](#)
- [Is Thermal Expansion Due to Particle Vibration? \(PDF: DOI\) \(中文: DOI\)](#)
- [Superfluids Are Not Fluids \(PDF: DOI\) \(中文: DOI\)](#)
- [Why a Phase Transition Temperature Remains Constant \(PDF: DOI\) \(中文: DOI\)](#)
- [What Causes Friction to Produce Heat? \(PDF: DOI\) \(中文: DOI\)](#)
- [The Easiest Way to Grasp Entropy \(PDF: DOI\) \(中文: DOI\)](#)
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- [Fundamental Problems about Mass \(PDF: DOI\) \(中文: DOI\)](#)
- [The Evolution from the Law of Gravitation to General Relativity \(PDF: DOI\) \(中文: DOI\)](#)
- [The Simplest Derivation of  \$E = mc^2\$  \(PDF: DOI\) \(中文: DOI\)](#)
- [How to Understand Relativity \(PDF: DOI\) \(中文: DOI\)](#)
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